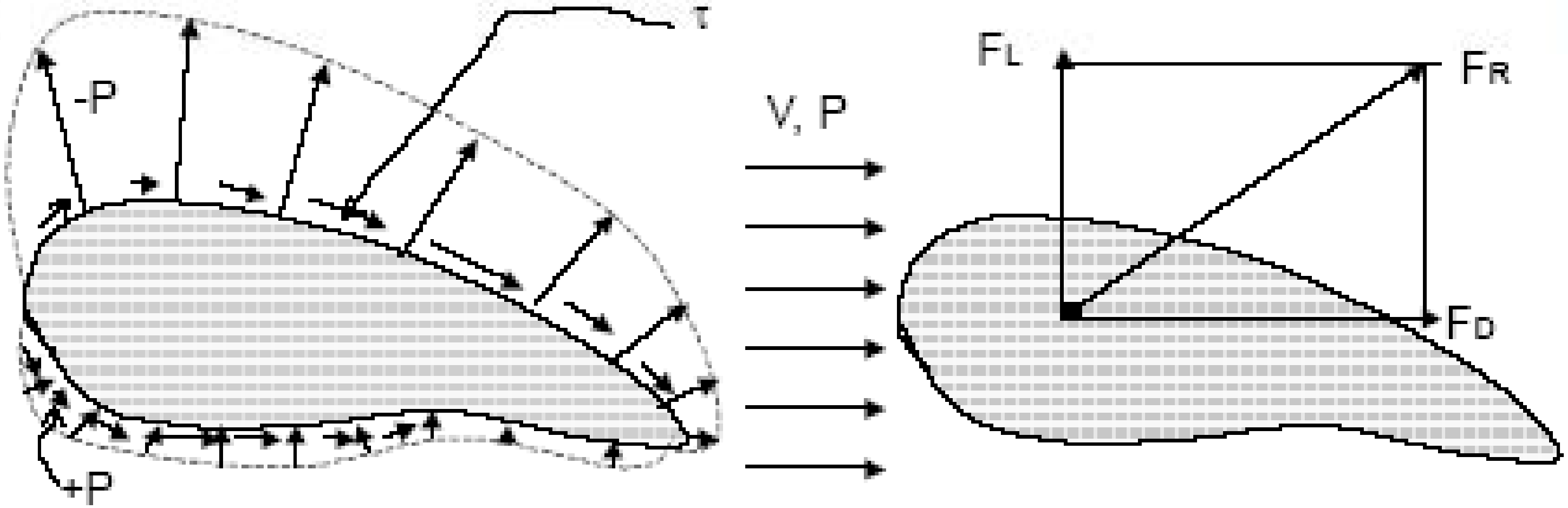
Aero- and Hydrodynamic Properties

- pneumatic conveying and separation
- use of water to transport bio-materials

Drag Coefficient



Flow about immersed object

Lift Force, $F_L = f_1 (A_P, \rho_f, V, \mu, E)$

Drag Force, $F_D = f_2 (A_P, \rho_f, V, \mu, E)$

$$F_L = C_L A_P \rho_f \frac{V^2}{2}$$

$$F_D = C_D A_P \rho_f \frac{V^2}{2}$$

C_L = lift coefficient

C_D = drag coefficient

A_P = projected area

ρ_f = fluid density

μ = fluid viscosity

V = velocity

Resultant Force

F_R = resistance drag force, N

$$F_R = \sqrt{F_D^2 + F_L^2}$$

C = overall drag coefficient

A_P = projected area normal to direction of motion, m^2

$$F_R = C A_P \rho_f \frac{V^2}{2}$$

V = relative velocity bet. main body of fluid and object, m/s

Laminar flow: fluid density variation is small

viscous action governs flow

profile or pressure drag is negligible

Turbulent flow: fluid compression governs flow

frictional drag is negligible

Frictional Drag

$C = f(Re, \ldots)$

Re = Reynolds number

d = effective dimension of the object, e.g. length of a plate, diameter of a sphere

$$Re = \frac{dV\rho_f}{\mu}$$

ρ_f = fluid density

μ = fluid viscosity

For flat plate with laminar boundary layer (Vennard, 1961; Prandtl, 1932):

$$C_f = \frac{1.328}{Re^{0.5}}$$

For flat plate with turbulent boundary layer (Vennard, 1961; Prandtl, 1932):

$$C_f = \frac{0.455}{(\log Re)^{2.58}}$$

For transition region, where laminar flow changes to turbulent flow (Prandtl, 1932):

$$C_f = \frac{0.455}{(\log Re)^{2.58}} - \frac{1700}{Re}$$

$$F_D = C_D A_P \rho_f \frac{V^2}{2}$$

← C_f can be substituted to this equation

For plate-like materials, F_D can be multiplied by 2 to account for two sides of the plate.

The equations for C_f are for frictional drag of smooth flat plates aligned with flow.

If the plate or circular disk is placed normal to the flow, the total drag will contain negligible frictional drag and does not change with Re.

Profile or Pressure Drag

Blunt objects, e.g. sphere, when placed in a fluid flow, frictional drag can be neglected because of the small surface area for friction to act.

Exception: when Re < 1, Stokes Law applies:

$$F_D = 3\pi\mu V d_p$$

F_D = Drag force

μ = fluid viscosity

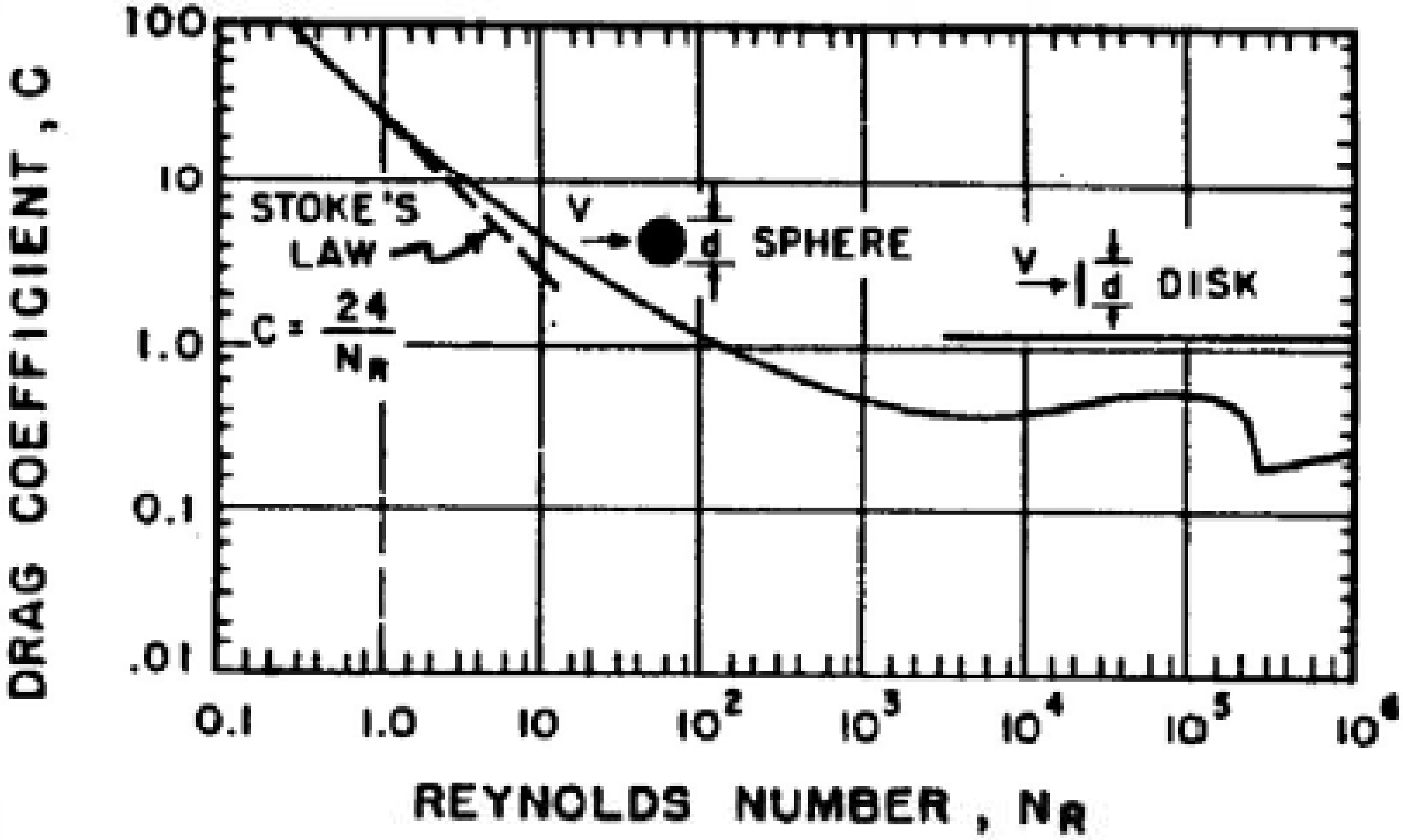
V = fluid velocity

d_p = diameter of spherical object

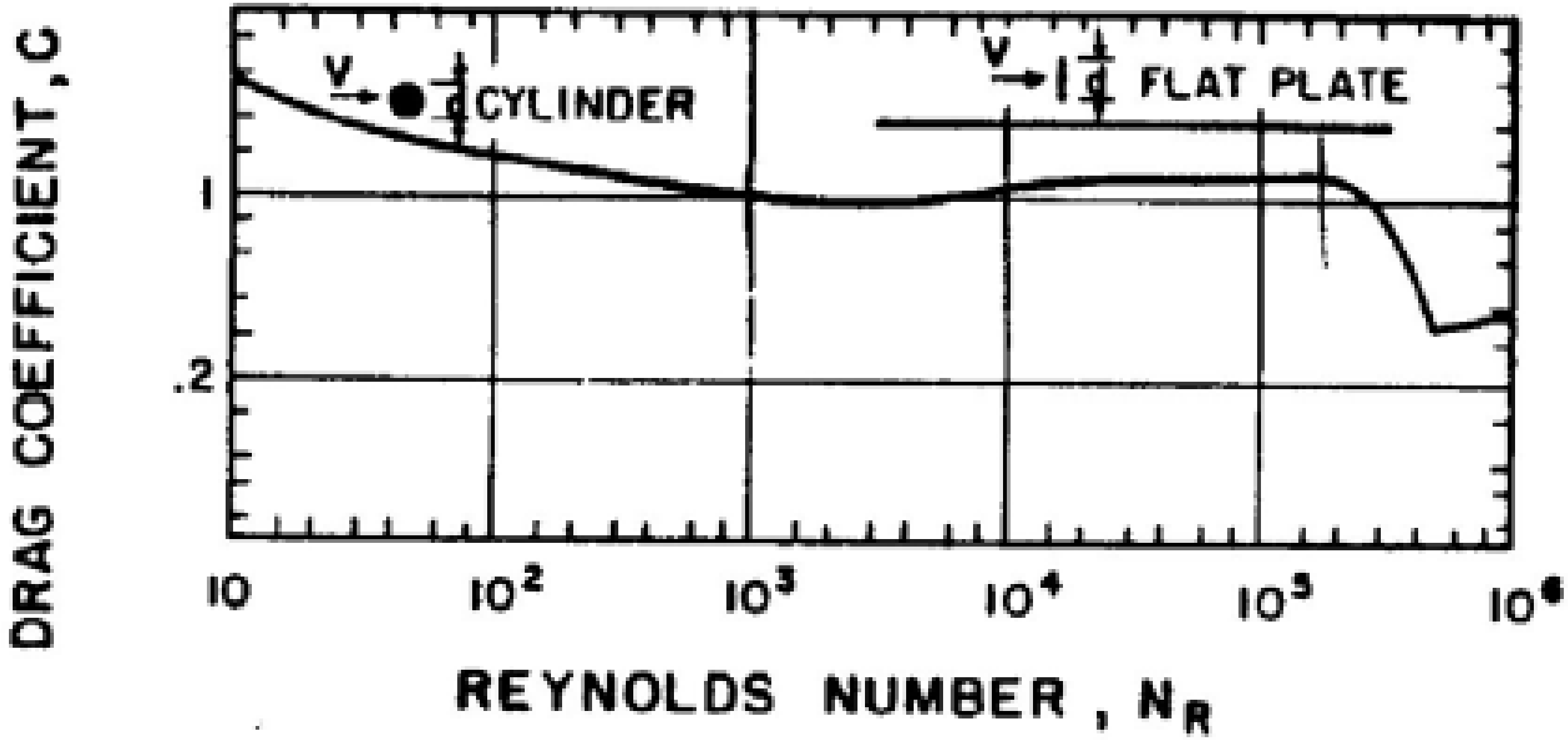
Equating the two equations for Drag Force, taking A_p to be the frontal area equal to $(\pi/4)d^2_p$

$$C_D\left[\frac{\pi}{4}d_p^2\right]\rho_f\,\frac{V^2}{2}=3\pi\mu V d_p$$

$$C_D=\frac{24}{Re}$$ ⟵ profile drag coefficient



Drag coefficient for spherical and disk-shape particles (Vennard, 1961)



Drag coefficient for cylindrical and flat plate particles (Vennard, 1961)

Terminal Velocity

During free fall, it is the velocity at which the net gravitational accelerating force, F_g equals the resisting upward drag force F_r.

Under steady state condition, where terminal velocity has been achieved:

$\rho_p > \rho_f$ - particle will move downward

$\rho_p < \rho_f$ - particle will move upward

$$F_g = F_r \quad \text{when} \quad V = V_t$$

substituting for F_g and F_r, the expression for terminal velocity will be:

$$m_p g \left[\frac{(\rho_p - \rho_f)}{\rho_p} \right] = C A_p \rho_f \frac{V_t^2}{2}$$

or

$$V_t = \left[\frac{2 m_p g (\rho_p - \rho_f)}{\rho_p \rho_f A_p C} \right]^{1/2}$$

and

$$C = \frac{2m_p g(\rho_p - \rho_f)}{V_t^2 \rho_p \rho_f A_p}$$

g = acceleration due to gravity, m/s^2

m_p = mass of particle, kg

ρ_p = mass density of particle, kg/m^3

ρ_f = mass density of fluid, kg/m^3

A_p = projected area of the particle normal to the motion, m^2

$C = C_f + C_D$

Laminar flow: C_f is generally negligible

Turbulent flow: C_f usually negligible except for streamlined bodies

Example: Derive the equation to determine the terminal velocity of a sphere.

Hint: $V_t = f(d_p, \rho_p, \rho_f, C)$

If assuming a laminar flow, then,

$V_t = f(d_p, \rho_p, \rho_f, \mu)$

For conditions of turbulent flow in the region where $10^3 < Re < 2x10^5$ and $C \approx 0.44$, the following equation for terminal velocity applies (Lapple, 1956):

$$V_t = 1.74 \left[\frac{g d_p (\rho_p - \rho_f)}{\rho_f} \right]^{1/2}$$

For an intermediate region of $2<Re<10^3$, the drag coefficient is given by:

$$C=\frac{18.5}{(Re)^{0.6}}$$

And the terminal velocity by:

$$V_t=\frac{0.153g^{0.714}d_p^{0.142}\left(\rho_p-\rho_f\right)^{0.714}}{\rho_f^{0.286}\mu^{0.428}}$$

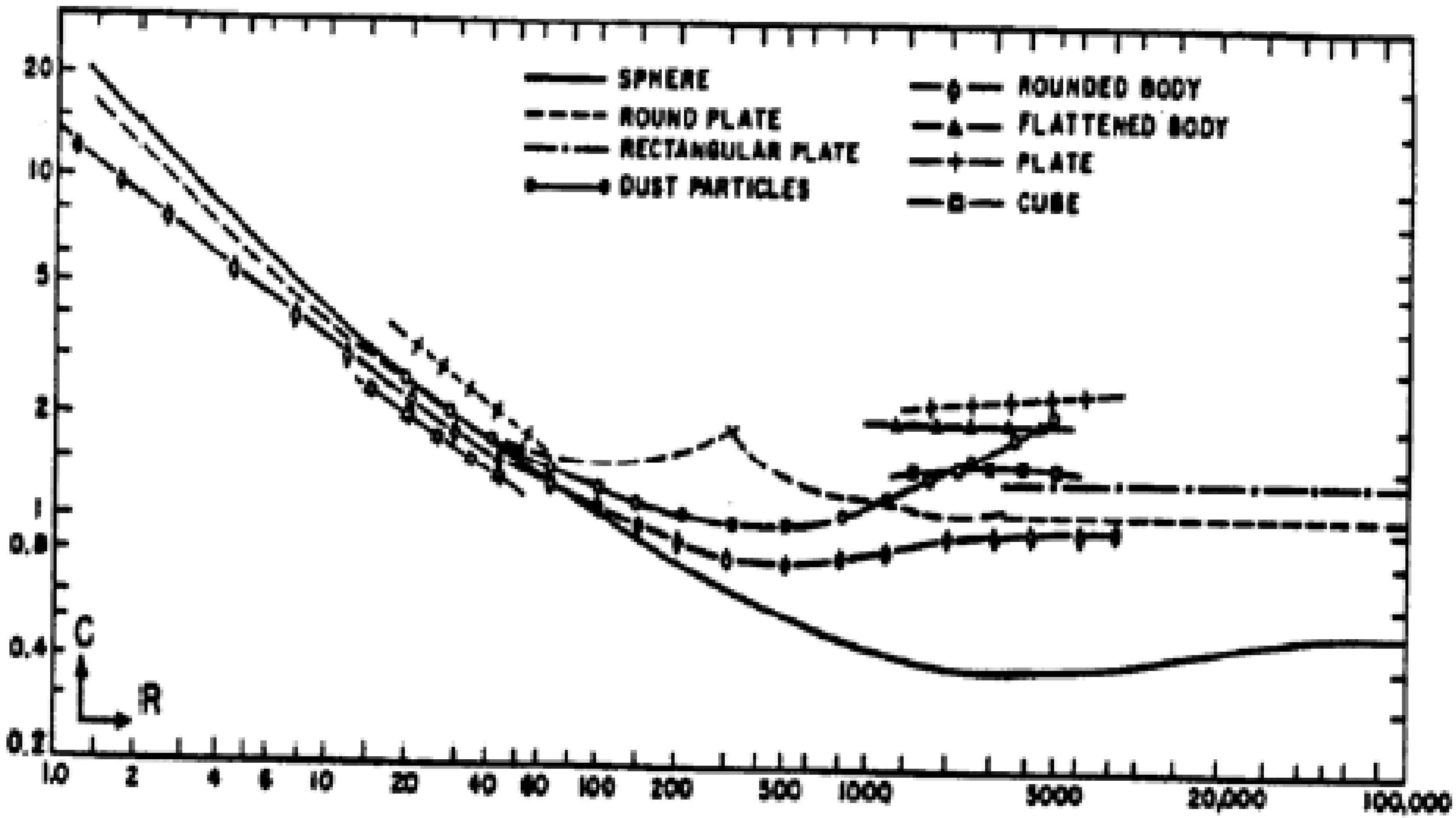
Table 9.1 Comparative summary of equations of motion of spheres, disks and circular cylinders (Adopted from Lapple, 1956)

	Sphere (any direction[1])	Thin disk (normal to face[1])	Thin disk (parallel to face[1])	Infinite circular cylinder (normal to axis[1])
Reynolds No. equ.	$d_p V \rho_f/\eta$	$d_p V \rho_f/\eta$	$2LV\rho_f/\eta$	$d_p V \rho_f/\eta$
Frontal area A_p	$(\pi/4)d_p^2$	$(\pi/4)d_p^2$	$(d_p)L$	$(d_p)K$
Mass m_p	$\rho_p(\pi/6)d_p^3$	$\rho_p(\pi/4)d_p^2 L$	$\rho_p(\pi/4)d_p^2 L$	$\rho_p(\pi/4)d_p^2 L$
Drag relationships				
streamline flow				
$N_R<0.2$, $F_D=$	$3\pi\eta V d_p$	$8\eta V d_p$	$(16/3)\eta V d_p$	$(4\pi/K)\eta VL$
$C_D N_R=$	24	$64/\pi$	64/3	$8\pi/K$
turbulent flow				
C_D (average)	0.44	1.12	—	1.2
N_R (range)	$1\times10^3-2\times10^5$	>1000	—	$1\times10^2-2\times10^5$
Terminal velocity V_t^2	$\frac{4gd_p(\rho_p-\rho_f)}{3C\rho_f}$	$\frac{2gL(\rho_p-\rho_f)}{C\rho_f}$	$\frac{gd_p(\rho_p-\rho_f)}{2C\rho_f}$	$\frac{gd_p(\rho_p-\rho_f)}{2C\rho_f}$

[1] Direction of flow or motion.

L = Thickness of disk, length of rod or cylinder, length of flat plate along direction of flow or motion

K = 2.002 ln N_R



Drag coefficient for geometric and non-geometric shapes

Terminal Velocity from Drag Coefficient-Reynolds Number Relationship

CRe^2 or C/Re are first calculated and plotted against Re

For spherical particles, combine the following:

$$Re = \frac{d_p V \rho_f}{\mu}$$

and

$$V_t = \left[\frac{4 g d_p (\rho_p - \rho_f)}{3 \rho_f C} \right]^{1/2}$$

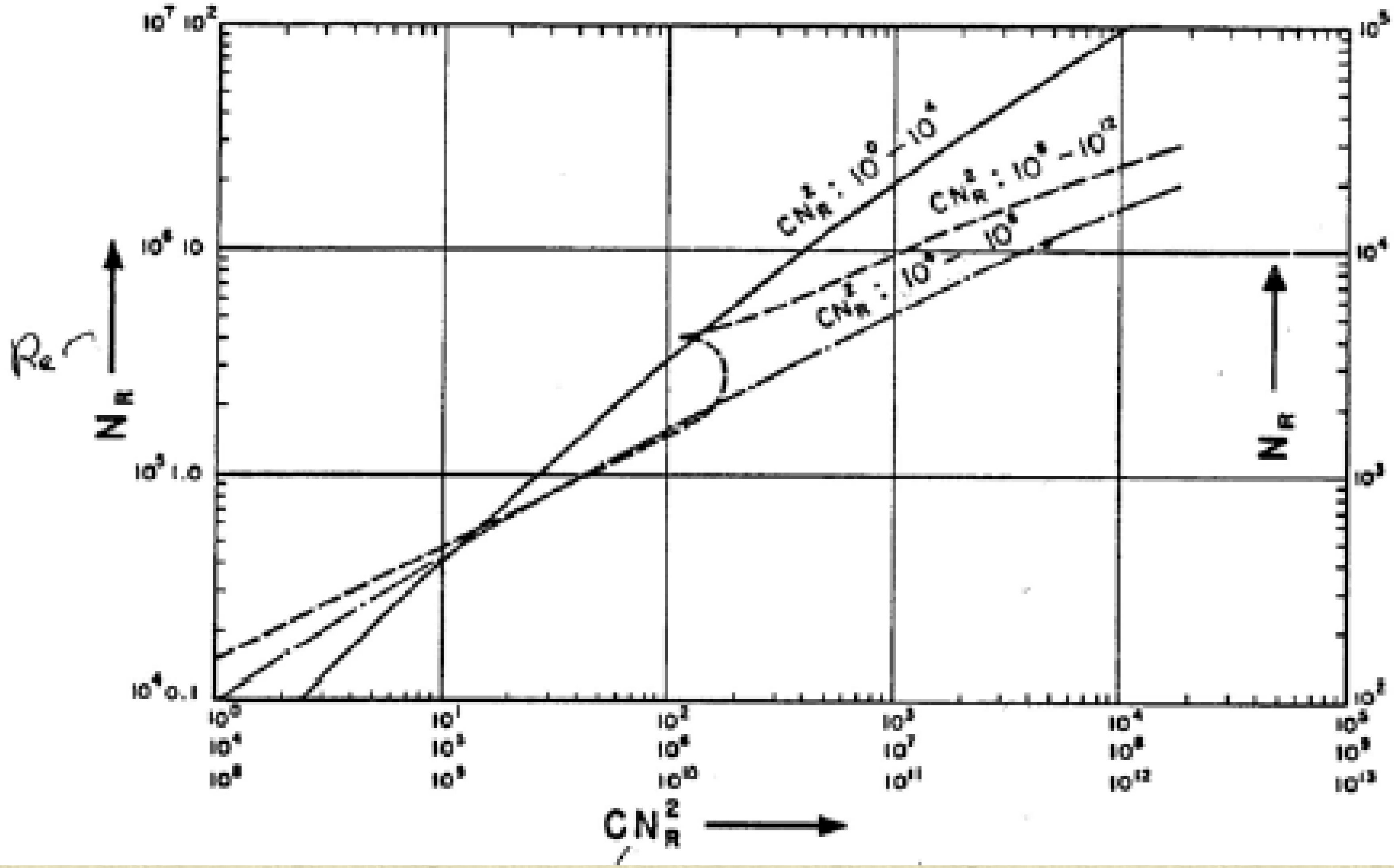
Terminal Velocity from Drag Coefficient-Reynolds Number Relationship

For spherical particles:

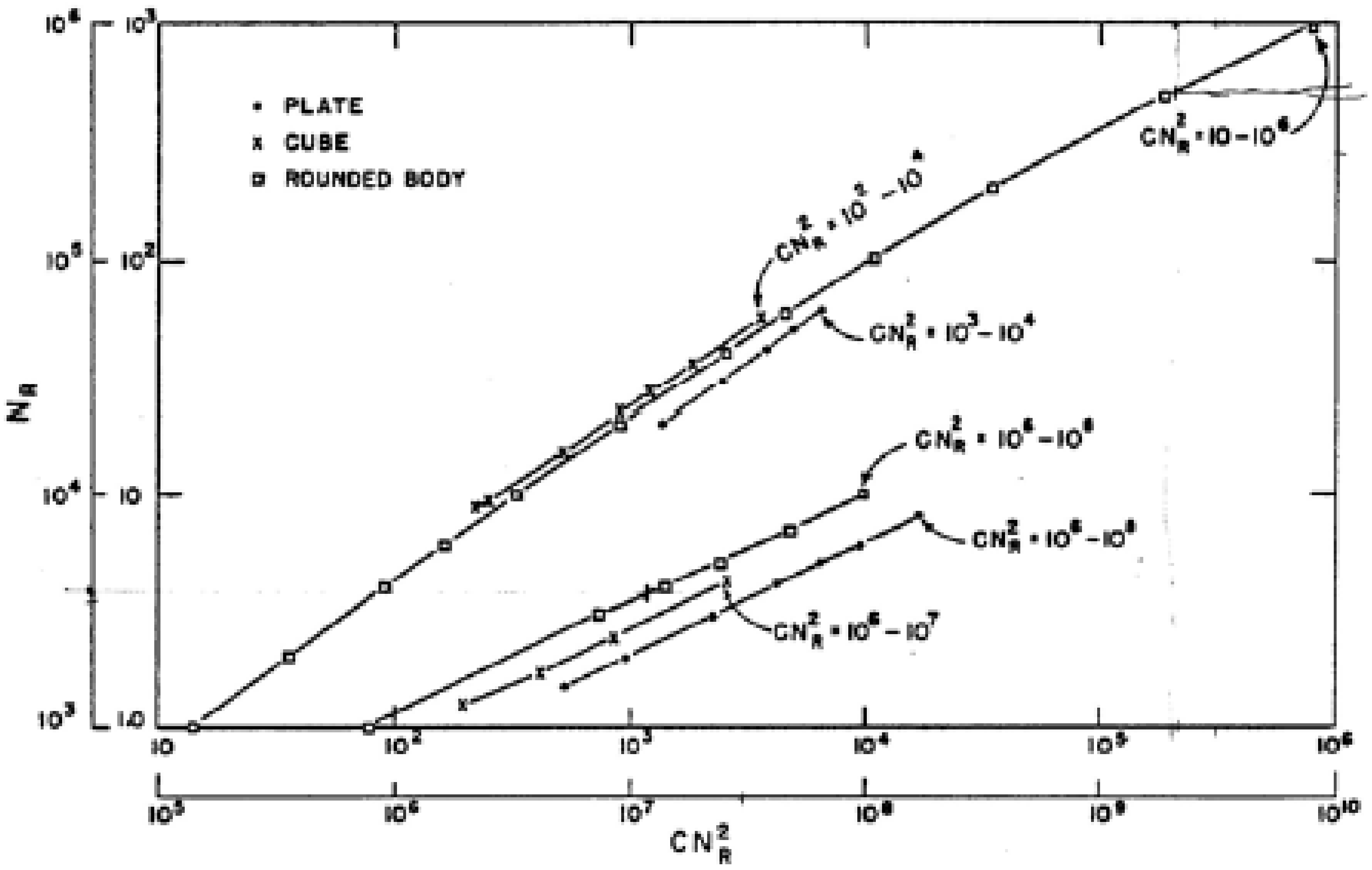
$$C\,Re^2 = \frac{4 g \rho_f d_p^3 (\rho_p - \rho_f)}{3\mu^2}$$

$$\frac{C}{Re} = \frac{4 g \mu (\rho_p - \rho_f)}{3 \rho_f^2 V_t^3}$$

$$C\,Re^2 = \frac{8 m_p g \rho_f (\rho_p - \rho_f)}{\pi \mu^2 \rho_p}$$



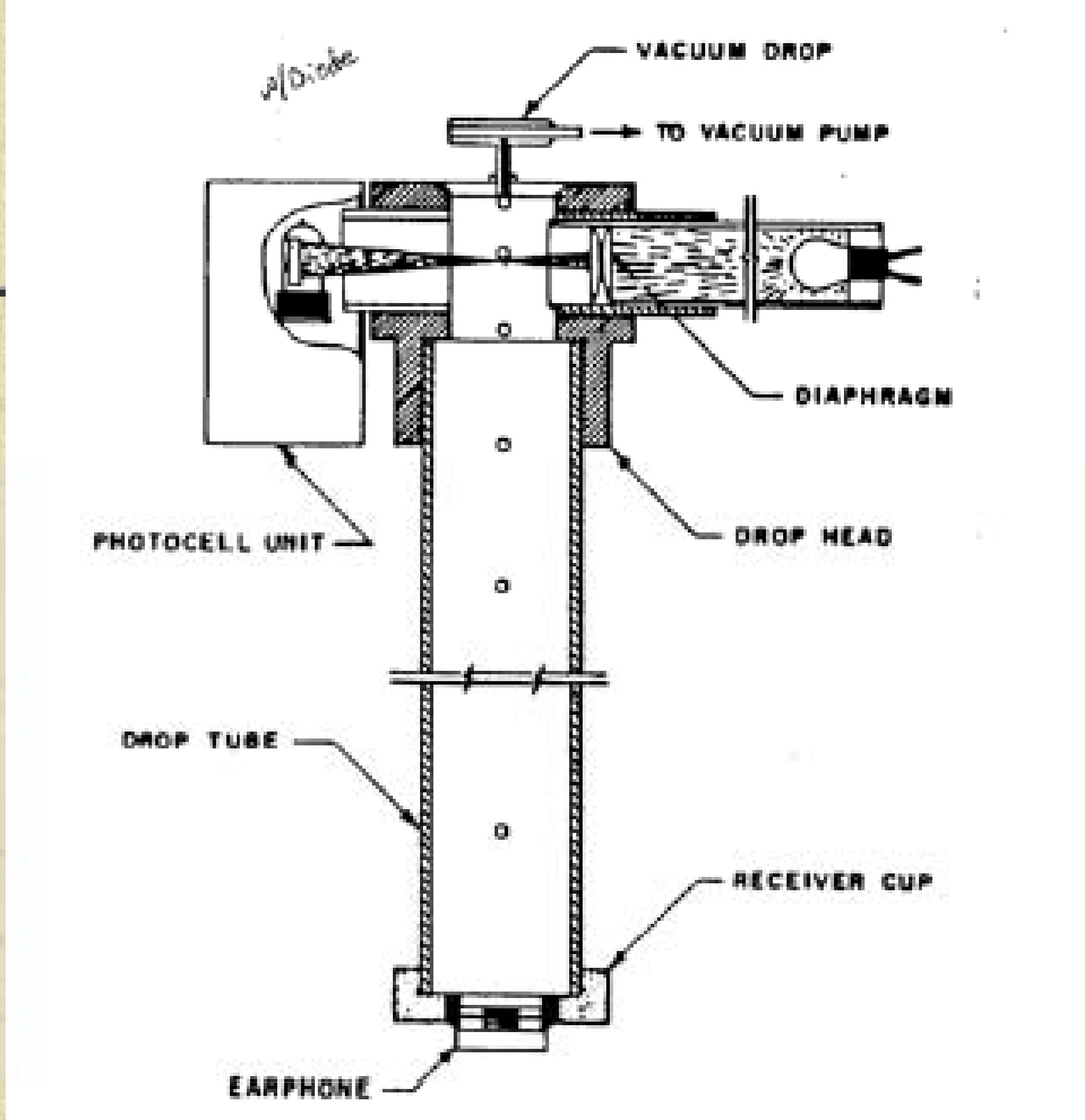
Reynolds number, Re vs. CRe^2 for spheres.



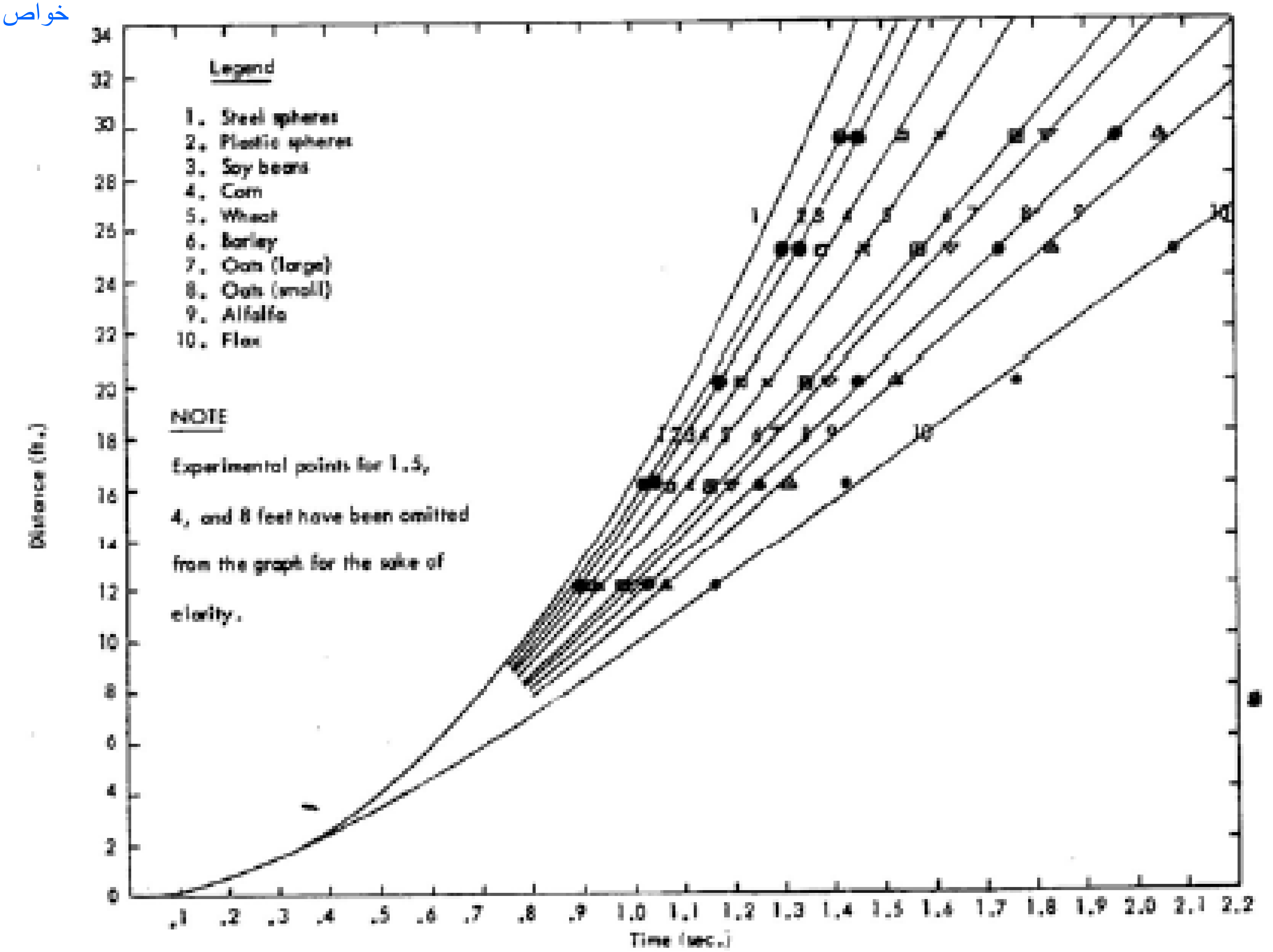
Reynolds number, Re vs. CRe² for plate-like, cube and rounded bodies.

Example: A single wheat kernel having a geometric mean diameter of 4 mm and density of 1400 kg/m^3 is dropped to freely fall into a water column at 21°C. Determine the terminal velocity of the grain kernel.

Example: To estimate the diameter of starch granules, their settling velocity in water at 21°C was found to be 0.1 mm/s. If the granule's density were 1500 kg/m^3, determine the average diameter of granules in mm. If these granules were falling in air, what would be their terminal velocity?



Drop tube for determining time-displacement curves for seeds and grains (Keck and Goss, 1965)



Displacement-time graph for seed grains (Bilanski et al. 1962)

Aerodynamic properties of grains (Bilanski et al. 1962)

Sample	Length (mm)	Width (mm)	Depth (mm)	Mass (mg)	Terminal velocity (m/s)	C	Re
Alfalfa	2.35	1.43	1.07	2.40	5.46	0.50	6.01
Flax	4.33	2.26	1.10	5.35	4.66	0.52	8.36
Wheat	6.95	3.35	2.96	45.36	8.99	0.50	27.2
Barley	8.81	3.20	2.38	33.11	7.01	0.50	22.8
Small oats	9.60	2.44	2.07	18.14	5.88	0.47	19.0
Large oats	12.22	2.80	2.16	33.57	6.34	0.51	24.8
Corn	11.64	8.02	4.15	285.77	10.64	0.56	57.7
Soybeans	7.77	6.77	5.88	205.93	13.51	0.45	62.8
Plastic spheres	9.51	9.51	9.51	526.18	16.77	0.43	108.5

Air column Method (Song and Litchfield 1991)

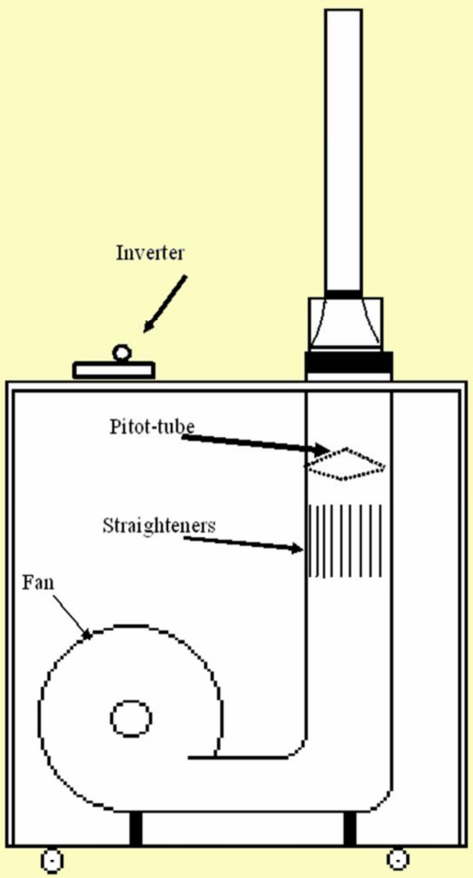


Figure.1- Schematic diagram of an air column used for measurement of terminal velocity.