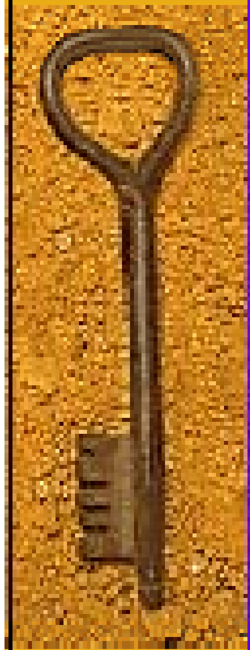
Friction and Flow of Granular Materials

Coefficient of friction – an important information for design of:

a) grain bins, silos and other storage structures

b) machinery, e.g. design of chopping and impelling unit need information on sliding coefficient of friction

c) conveyors, chutes, and other handling devices

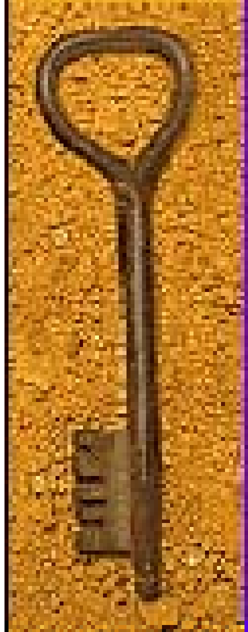


Basic Definitions and Properties

Forces of *static friction* - frictional forces acting between surfaces at rest with respect to each other. Forces necessary to start motion.

Forces of *kinetic friction* – friction forces existing between the surfaces in relative motion.

$$\mu = \frac{F_t}{F_n}$$ ← coefficient of friction



F_t = tangential force

F_n = normal force

μ_s = static coefficient of friction

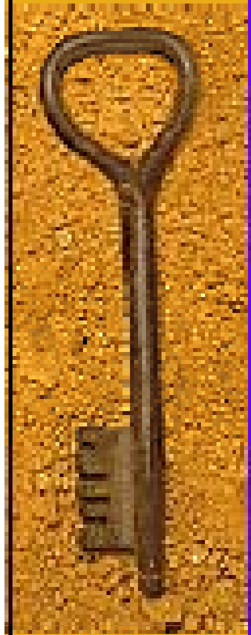
μ_d = dynamic coefficient of friction

$$\mu_s \geq \mu_d$$

Laws of Friction (verified by Coulomb).

Frictional force:

1. Is proportional to the normal load.
2. Is independent of the area of the sliding surfaces.
3. Is largely independent of sliding velocity.
4. Depends upon the nature of the material in contact.

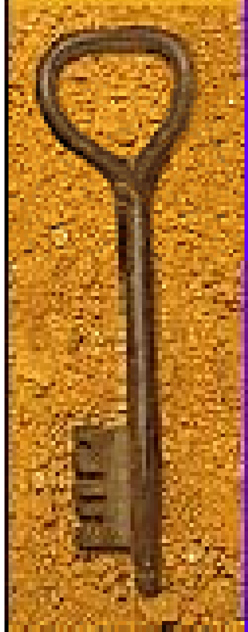


More commonly accepted concepts of friction (Sherwood 1951)

The friction force:

1. May be defined as the force acting in a plane containing the contact point/s in such as manner as to resist relative motion of the contact surfaces.

2. May be regarded as being composed of two main components, a force required to deform and sometimes shear the asperities of the contacting surfaces, and a force required to overcome adhesion or cohesion between surfaces.



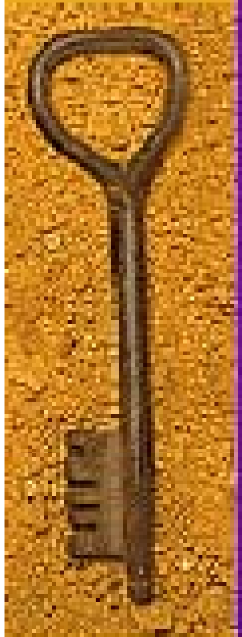
The friction force:

3. Is directly proportional to the actual contact area.

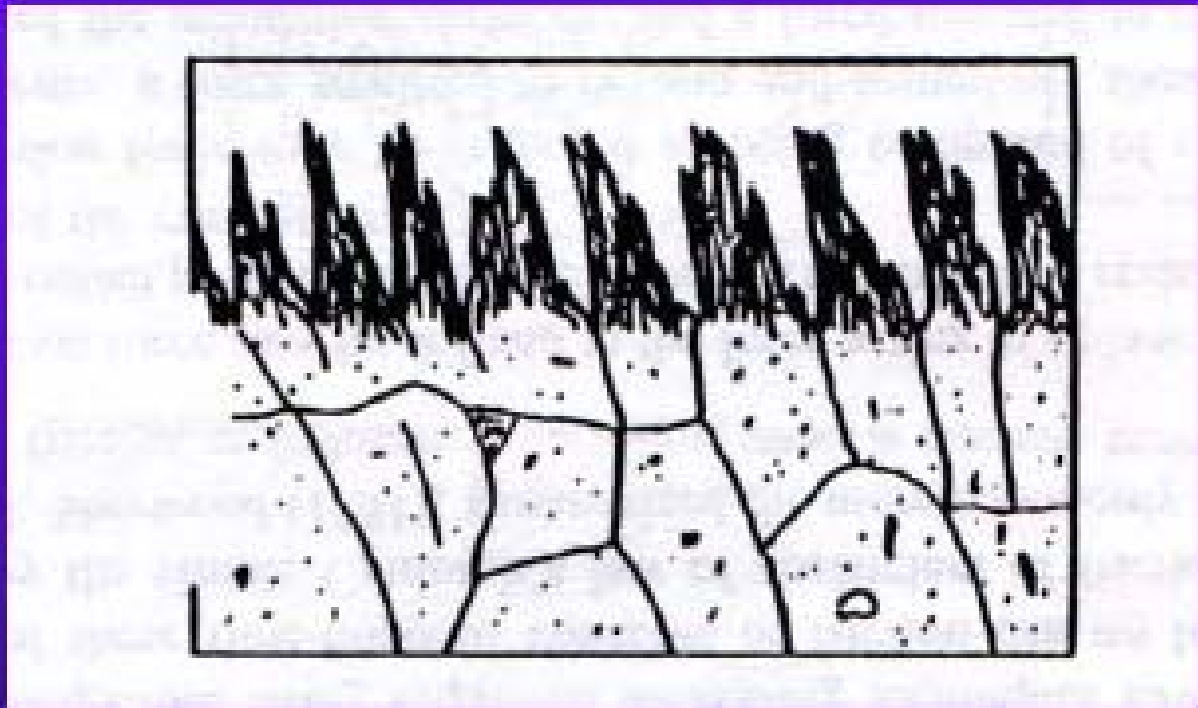
4. Depends on the sliding velocity of the contacting surfaces because of the effect of the velocity on the temperature of the contacting materials.

5. Depends on the nature of materials in contact.

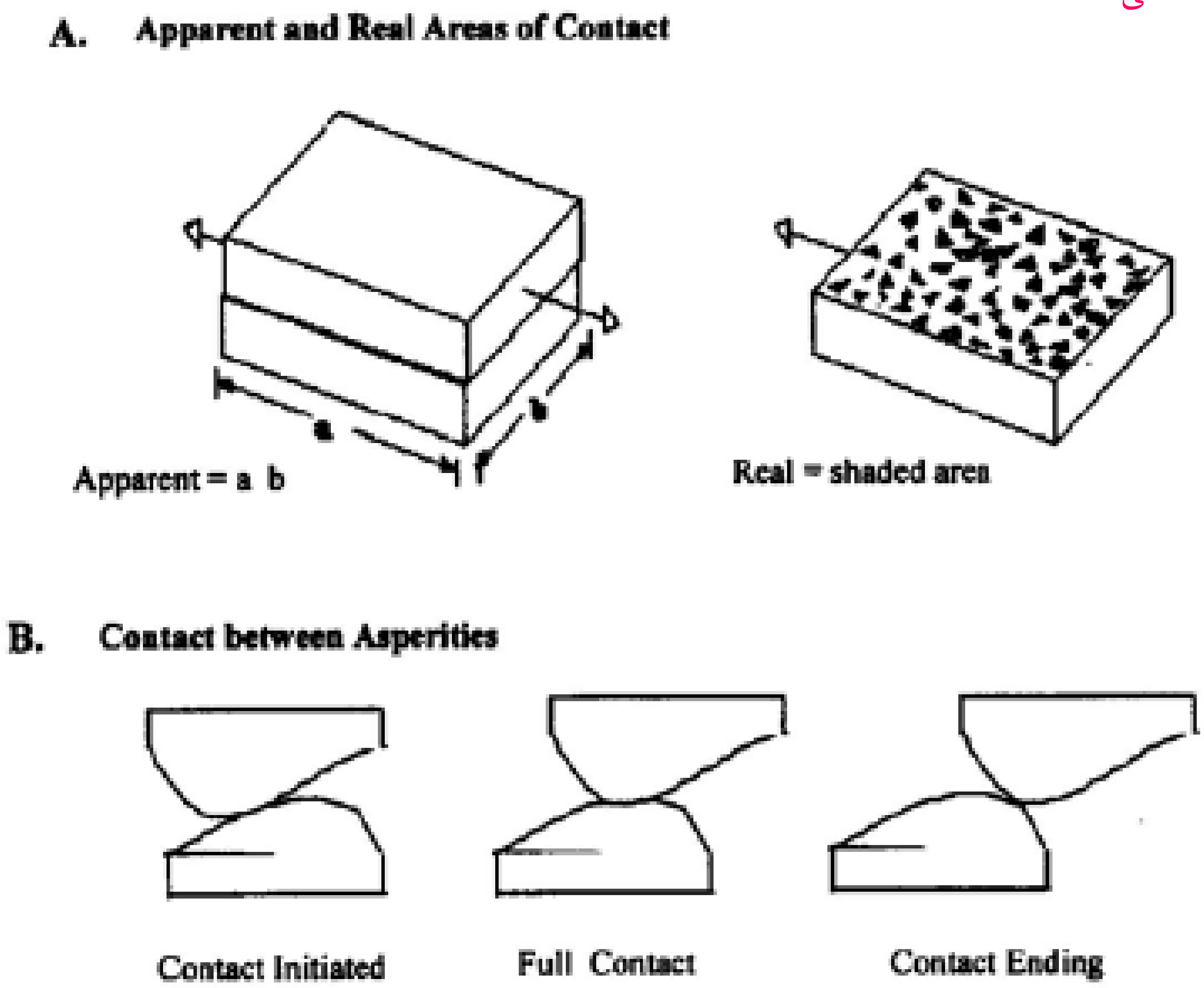
6. Is not dependent on the surface roughness except in the extremes on very fine and very rough surfaces.



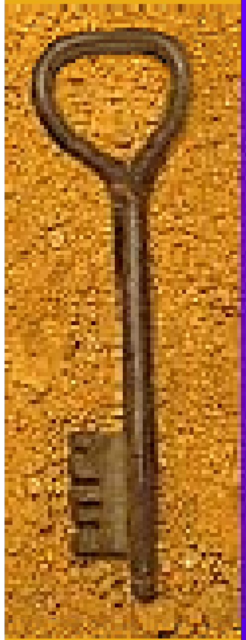
Asperity contact - even carefully prepared surfaces contain hills and valleys microscopically.



Asperities on a cross-section of finely turned copper surface (Bowden and Tabor 1956).



Microscopic aspects of friction of solids.



Angle of repose - angle with the horizontal at which the material will stand when piled.

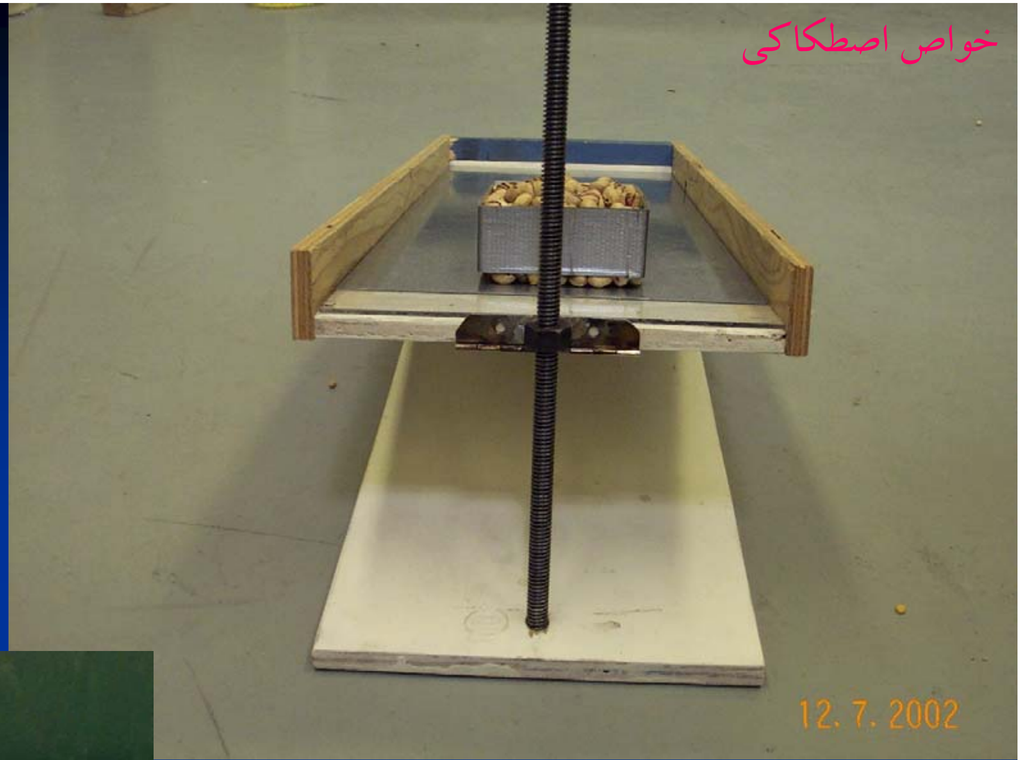
coefficient of friction between granular materials = tangent of the angle of internal friction for that material

Static angle of repose - angle of friction taken up by a granular solid about to slide upon itself. Filling angle of repose

Dynamic angle of repose - angle formed when a material empties from a bin. Emptying angle of repose

Tilting Table

خواص اصطکاکی



خواص اصطکاکی

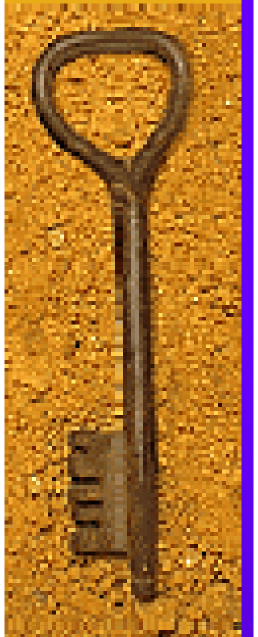


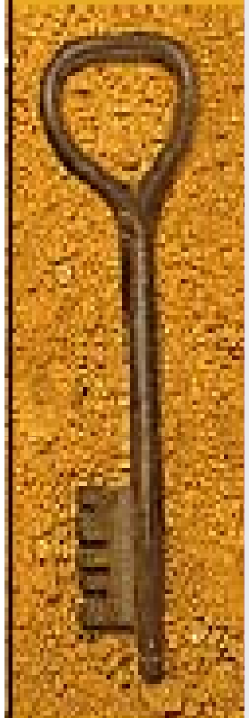
Table 9.2. Dynamic (sliding) Coefficients of Friction for Agricultural Materials.

Material	Type Surface & Characteristics	Moisture Content	Coef.	Reference
Alfalfa Pellets	Steel	n.r. [a]	0.17	Kososki (1965)
	Wood	n.r.	0.28	"
Alfalfa, Chopped	Steel	n.r.	0.34	Kososki (1965)
	Wood	n.r.	0.37	"
Corn Silage	Polished Galv. Steel[b], 426 Pa	73%	0.70	Richter (1954)
	Polished Galv. Steel, 680 Pa	73%	0.68	"
	Polished Galv. Steel, 1360 Pa	73%	0.66	"
Fish Meal	Steel	n.r.	0.35	Kososki (1965)
Limestone	Steel	n.r.	0.43	Kososki (1965)
Oyster Shells	Steel	n.r.	0.35	Kososki (1965)
Straw	Polished Galv. Steel[b], 680 Pa	n.r.	0.30	Richter (1954)
	Polished Galv. Steel, 1360 Pa		0.30	"

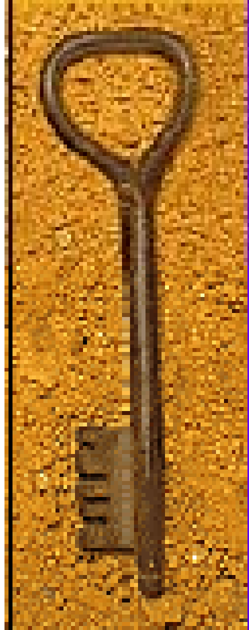
خواص اصطکاکی

Sample	Moisture (% w.b.)	Dockage (%)	Bulk density[1] (kg/m^3)	Filling angle of repose (°)		Emptying angle of repose (°)	
				Uncleaned	Cleaned	Uncleaned	Cleaned
Canary Seed var. 'Keith'	8.5	n/a	704.6	n/a	17.4	n/a	17.1
Caraway Seed	8.3	n/a	448.4	n/a	38.1	n/a	37.0
Chickpeas var. 'Desi'	9.9	n/a	779.8	n/a	28.9	n/a	29.0
Fenugreek Seed	10.1	n/a	753.0	n/a	30.0	n/a	30.3
Hemp var. 'Secuieni - 1'	6.17	32.07	370.7	37.53	n/a	36.16	n/a
Lentil var. 'Eston'	11.1	n/a	824.8	n/a	27.0	n/a	25.9
Mustard, brown	7.0	n/a	693.5	n/a	24.6	n/a	25.8
Mustard, oriental var. 'Cutlass'	6.4	n/a	674.0	n/a	25.2	n/a	27.0
Mustard, yellow var. 'Tilney'	7.3	n/a	726.9	n/a	24.9	n/a	26.0
Pea, green var. 'Ascona'	14.2	2.3	814.8	28.0	28.1	28.4	28.7
Pea, green var. 'Espace'	13.8	5.6	781.3	30.1	29.1	30.8	29.5
Pea, maple var. 'Setchey'	13.3	0.9	801.2	31.6	33.0	31.0	30.8
Yellow Pea var. 'Carneval' (Wetaskiwin1)	11.5	3.0	814.2	27.0	26.0	29.7	25.6
Yellow Pea var. 'Carneval' (Wetaskiwin2)	12.7	6.1	802.9	29.2	28.1	33.5	27.2
Safflower var. 'Saffire'	6.3	n/a	534.6	n/a	29.5	n/a	28.5

[1] Cleaned samples



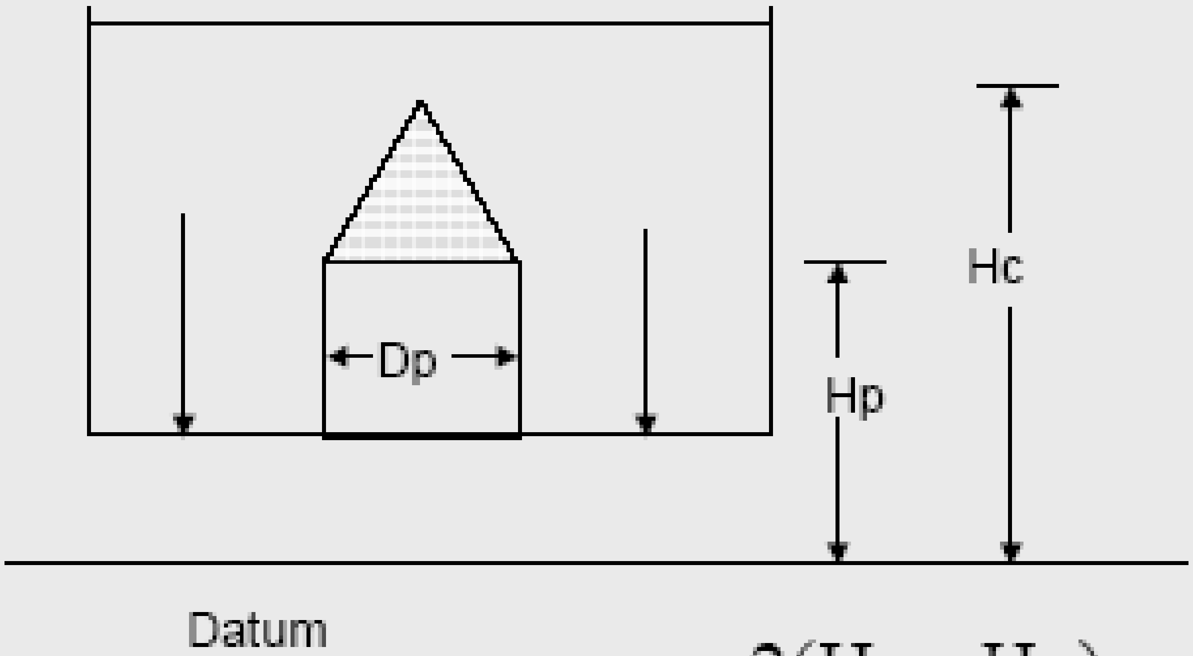
Angle of Repose of Specialty Crops



Measurement of angle of repose:

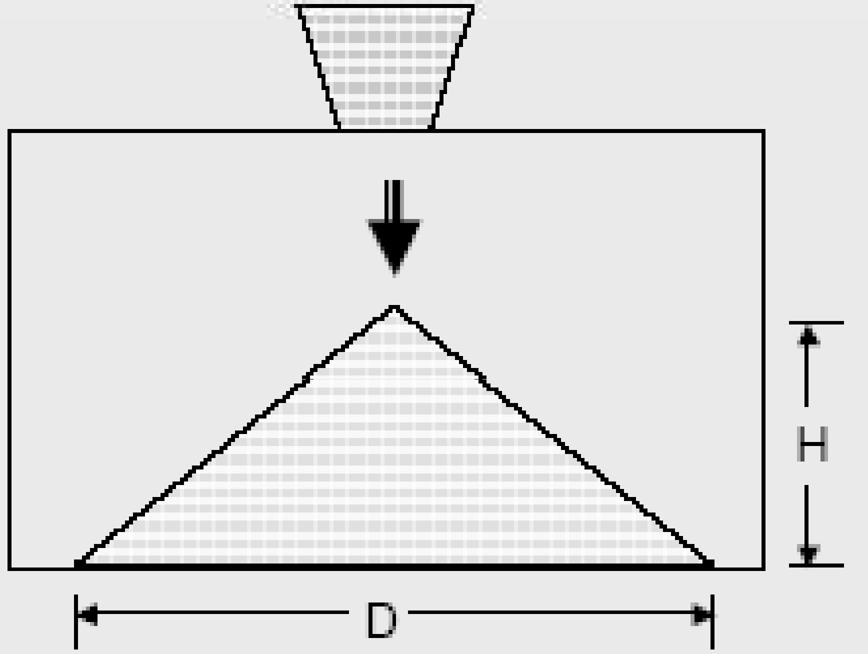
1. Tilting table

2. Immersed platform



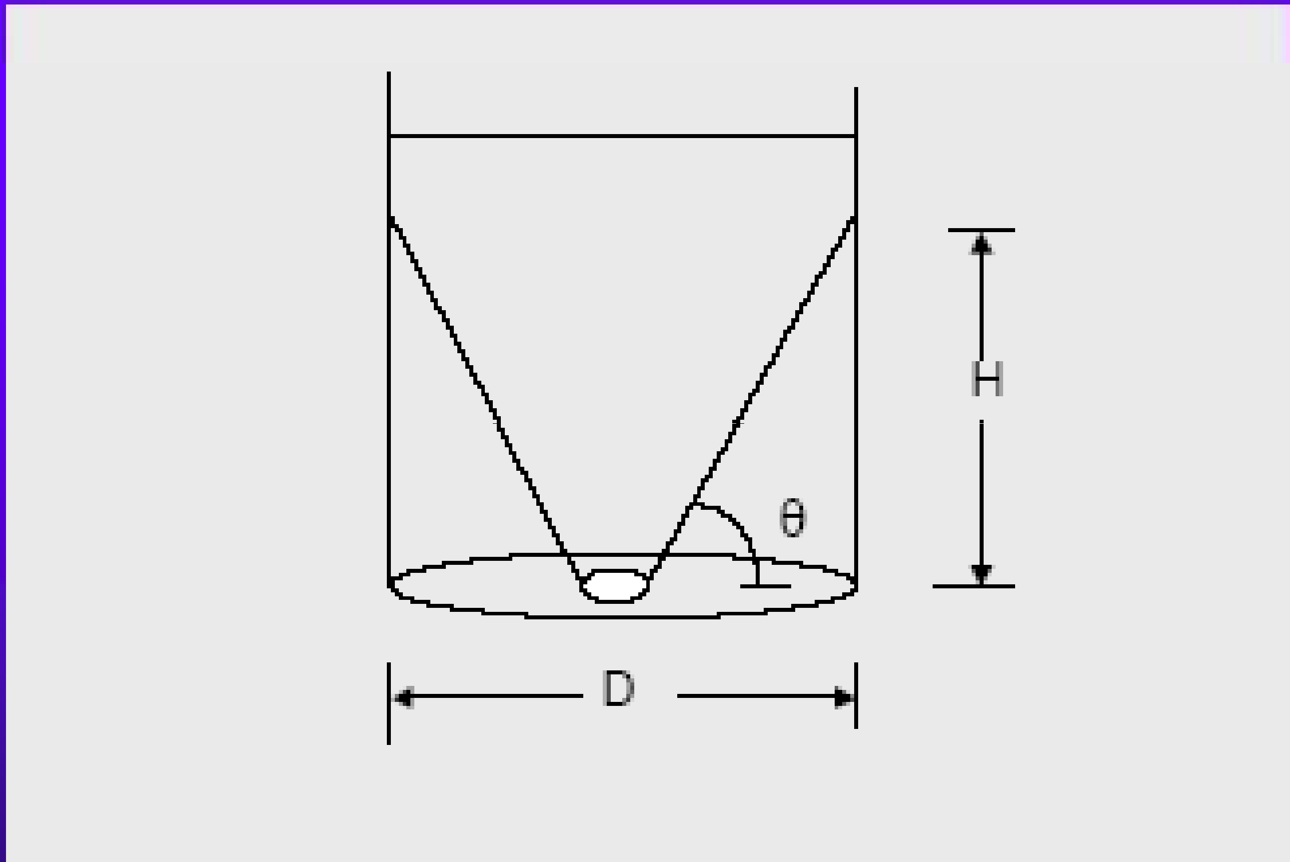
$$\theta = \tan^{-1}\frac{2(H_c - H_p)}{D_p}$$

3. Top Filling Box



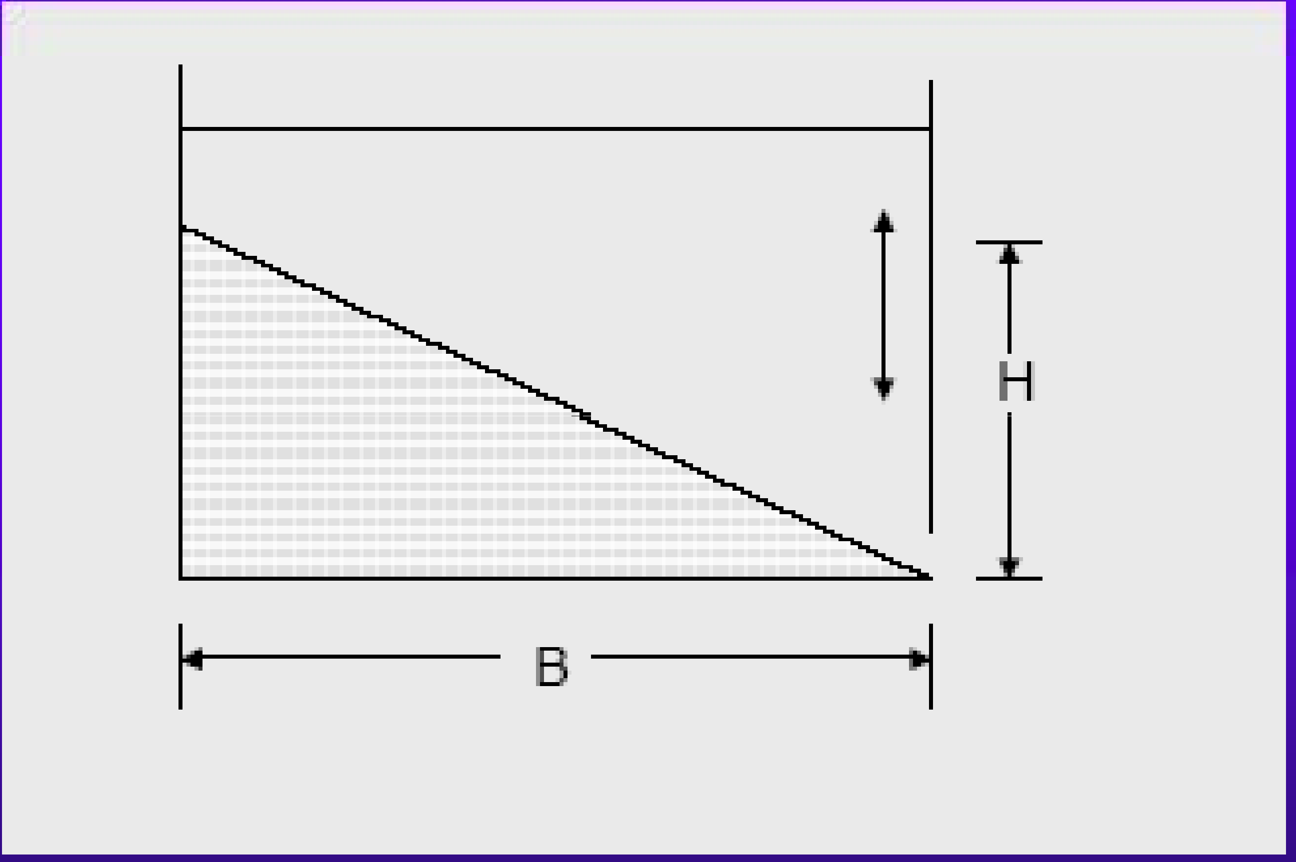
$$\theta = \tan^{-1} \frac{2H}{D}$$

4. Cylindrical Bin with a Hole

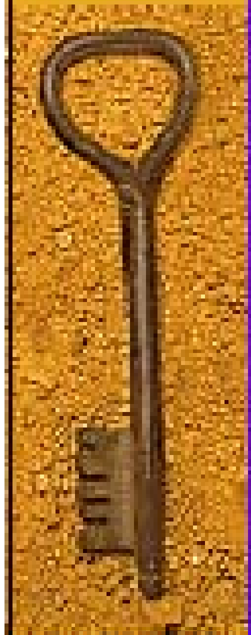


$$\theta = \tan^{-1}\frac{2H}{D}$$

5. Box with removable side



$$\theta = \tan^{-1}\frac{H}{B}$$

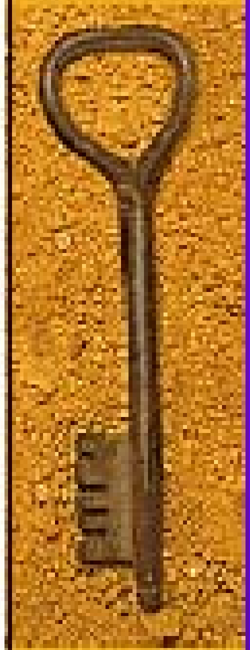


Angle of Internal Friction

- needed in the design of retaining walls, storage bins and hoppers
- quantifies resistance to motion

Rankine equation for the design of shallow bins:

$$\sigma_3 = \rho g y \tan^2\left(45 - \Phi_i/2\right)$$



σ_3 = lateral pressure against the wall at a point, Pa

y = distance from the top of the wall to where σ_3 is acting, m

ρ = bulk density, kg/m^3

Φ_i = angle of internal friction

Design of deep bins and similar storage structures, the pressure ratio k is needed.

$$k = \frac{\sigma_3}{\sigma_1} = \frac{1 - \sin \Phi_i}{1 + \sin \Phi_i}$$

where:

σ_3 = lateral pressure

σ_1 = vertical pressure

Φ_i = angle of internal friction

Janssen's equation for lateral pressure, σ_3:

$$\sigma_3 = \frac{\rho g R}{\mu_s}\left(1 - e^{-\frac{k\mu_s h}{R}}\right)$$

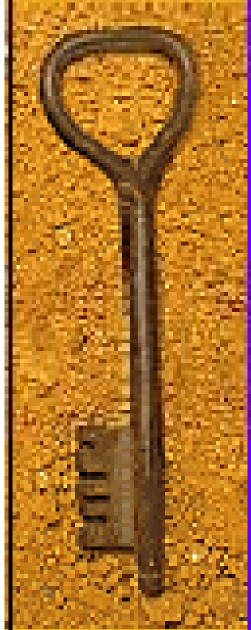
where

R = hydraulic radius or the ratio of cross sectional area to circumference, m^2/m

ρ = bulk density of material, kg/m^3

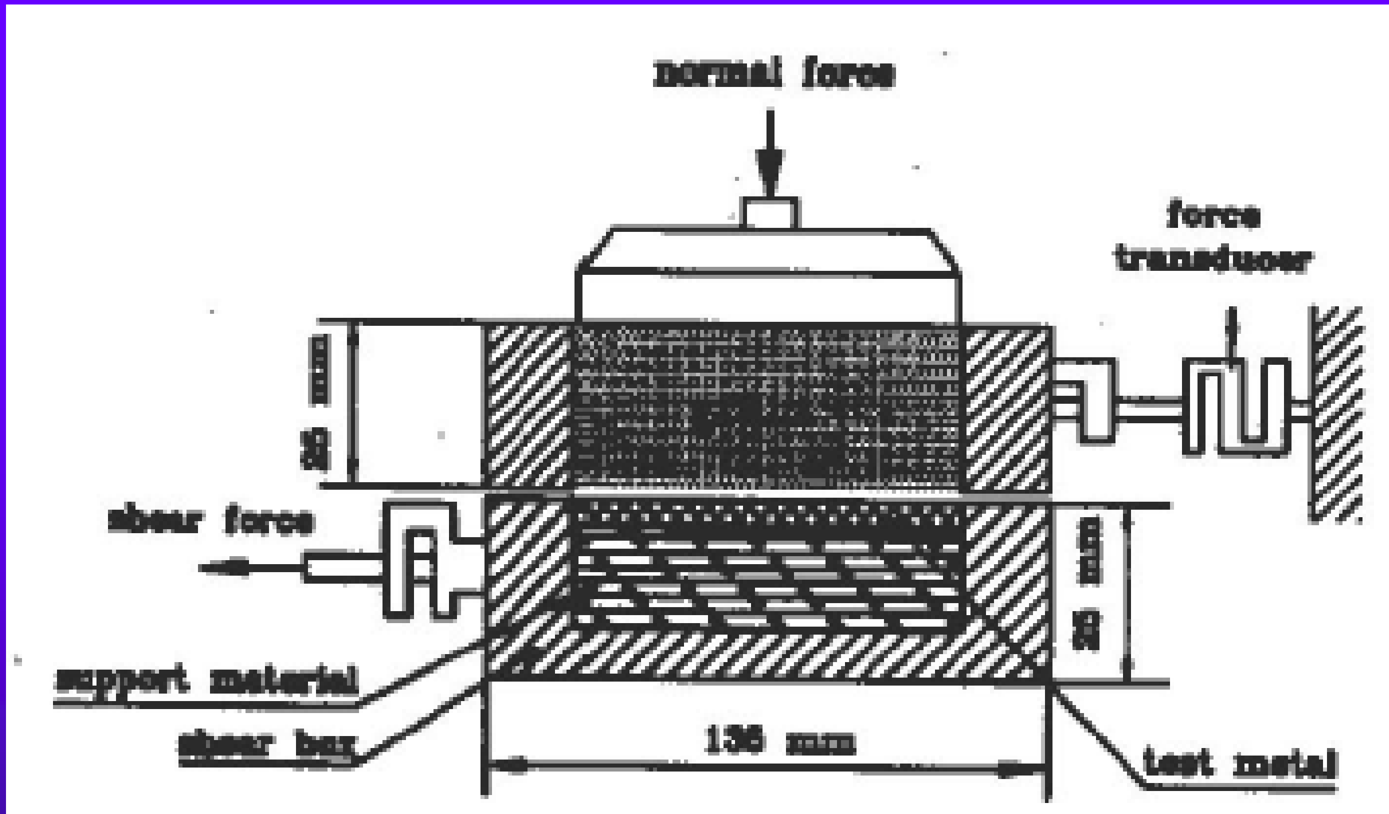
μ_s = static coefficient of friction of material against the wall

h = depth of material, m



Effect of moisture content of wheat on various components of Janssen's equation (Lorensen 1957)

M.C. (%)	k	μ_s	ρ	Repose Φ_r	Φ_i
7.3	0.463	0.453	790	29.6	23.5
11.0	0.420	0.432	790	29.3	24.5
14.1	0.357	0.433	756	31.0	26.5
17.1	0.280	0.471	728	35.6	27.3
19.3	0.310	0.592	704	41.0	23.2



Shear Box Apparatus to measure Angle of Internal Friction

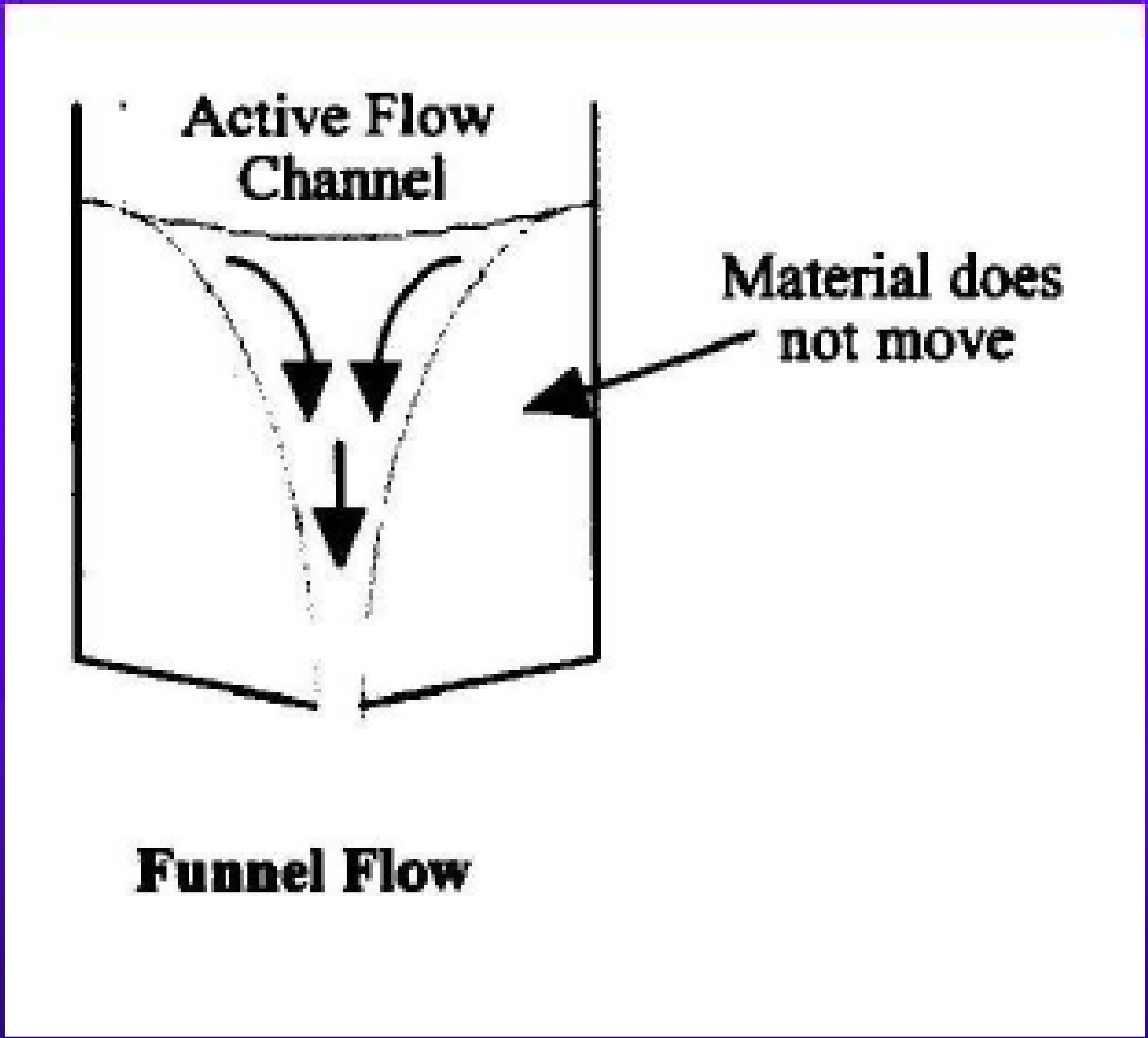
Table 9.4. Angle of internal friction and density of agricultural materials and food products.

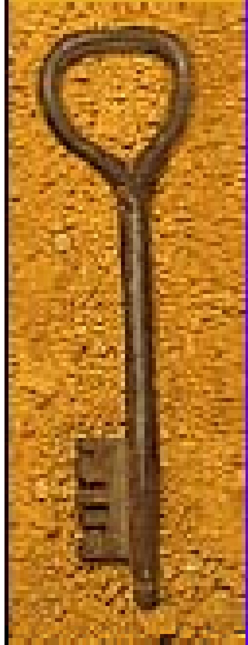
Material	Moist. Content (% w.b.)	Angle of Internal Friction	Bulk Density (kg/m^3)	Method of Measurement	Reference
Flour (Wheat)	10.6	29.6	842.	Direct Shear	Kamanth et al. (1991)
Milk (non-fat, Powdered)	2.7	26.	507.	Direct Shear	Hayashi et al. (1968)
	4.2	33.5	671.	"	"
	4.8	36.0	772.	"	"
Sand	n.r.	24.1	1730.	"	"
Sorghum	13.0	25.5	801.	Triaxial Test	Stewart (1968)
	17.7	26.0	801.	"	"
Sugar	0.014	34.0	1025.	Direct Shear	Hayashi et al. (1968)
Wheat	11.0	24.5	790.	?	Lorenzen (1957)
	17.1	27.3	727.	?	"

Flow of Granular Materials

- manner of flow from containers
- rate of flow through an orifice of given size

Example of free flowing granular material: dry, clean whole grain and pellet feeds


Active Flow
Channel
Material does
not move
Funnel Flow

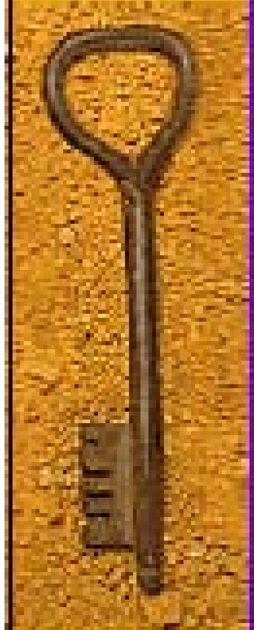


Funnel Flow

- occurs in bins with flat bottom or bins with sloping bottoms in which the angle it makes with the vertical is less than a critical angle (hopper angle).

- material first flows from the centre “core” of approx. size as the opening; flow expands in diameter like a funnel with height above the opening

- a ring of material supported by the floor remains stagnant



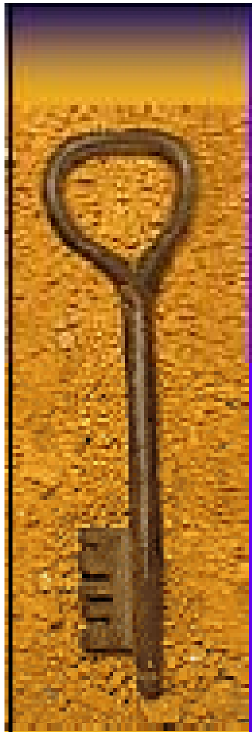
Funnel Flow

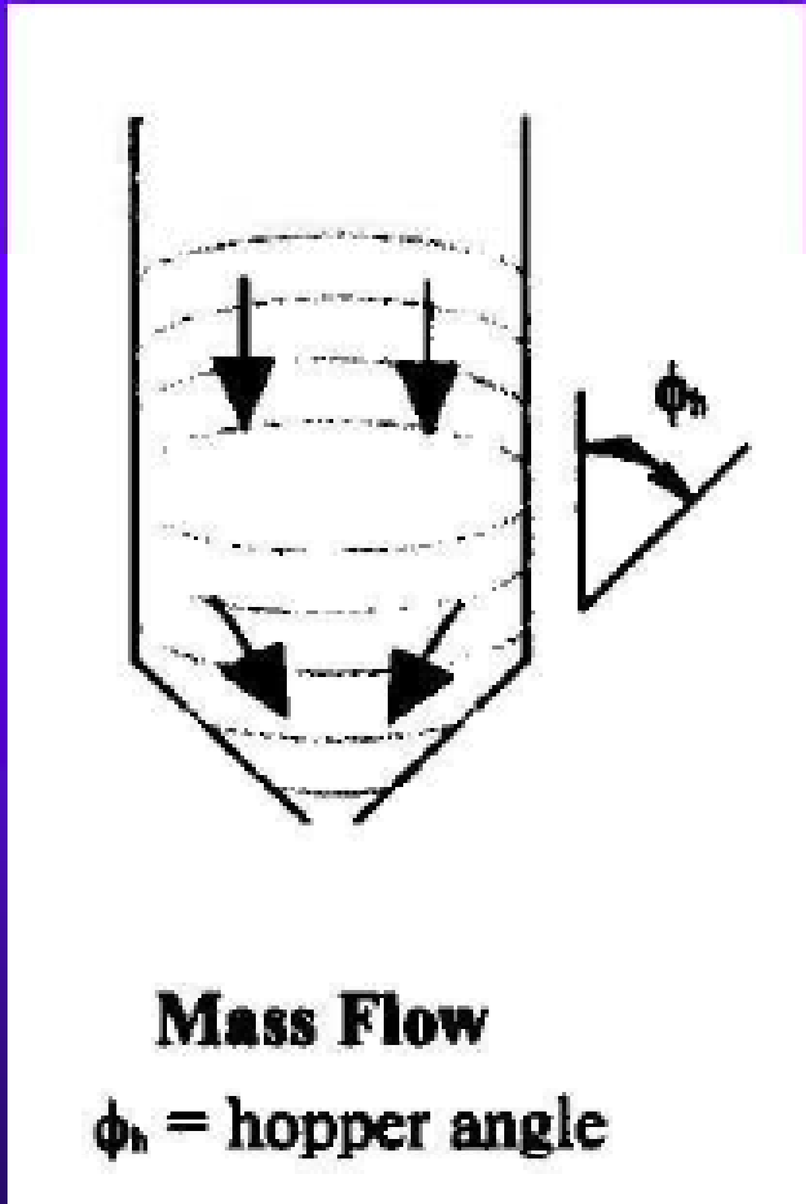
- as flow proceeds, material from the top surface flows into the “funnel” of flowing particles.

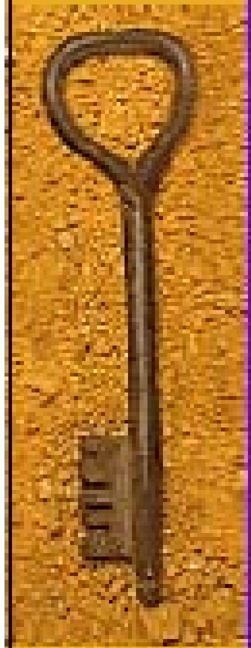
- simple to design and easy to build

- first material placed in the bin is the last removed

- stagnant material in the bin may remain there for excessive period of time; caking of this material may occur; erratic flow may occur




ϕh
Mass Flow
ϕh = hopper angle



Mass Flow

- all of the material flows towards the exit at the same time, ideally, at the same rate

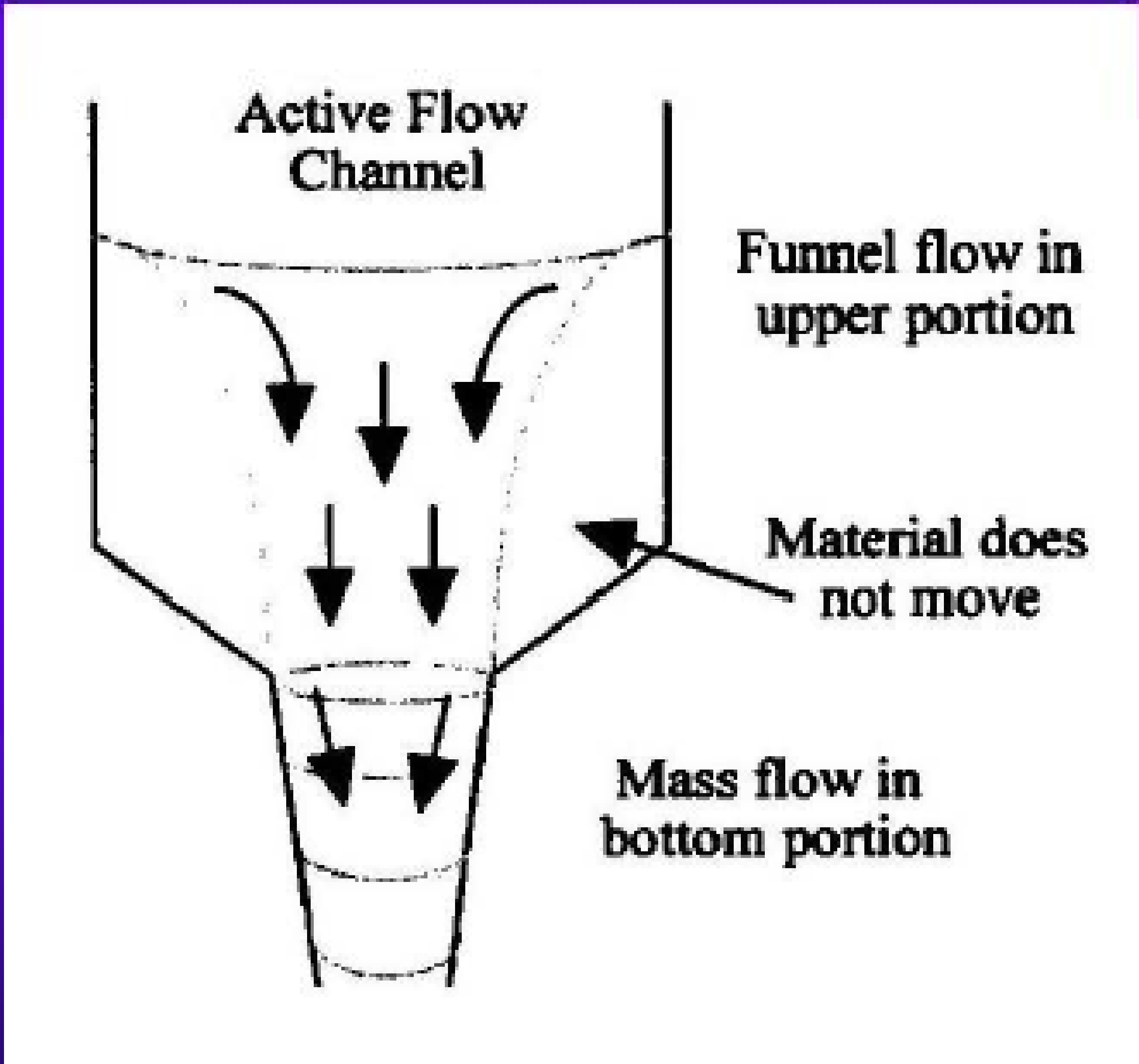
- suited for processing facilities where additional materials are continually added to the storage bin

- commonly used for feed storage in livestock facilties

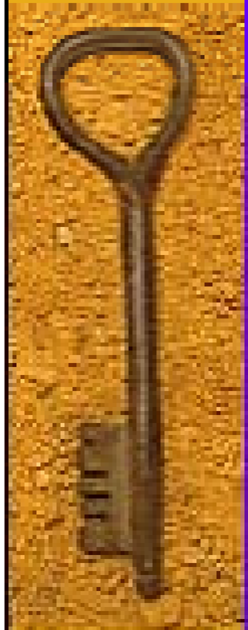
- flow is uniform

- must be higher to accommodate sloping bottom

- mass flow is achieved if the hopper angle ϕ_h is less than a specified value



Expanded Flow

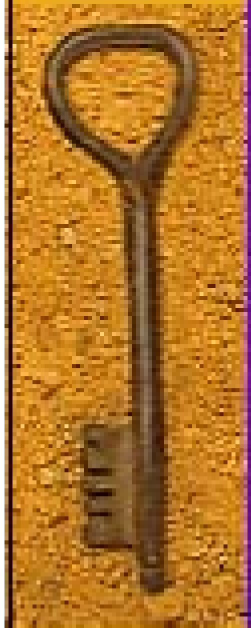


Expanded Flow

- constructing the bin with 2 regions having different slopes; mass flow is achieved in the larger portion of the bin

- modifying existing bins with funnel flow is possible

- more expensive and higher construction cost



Flow Rates

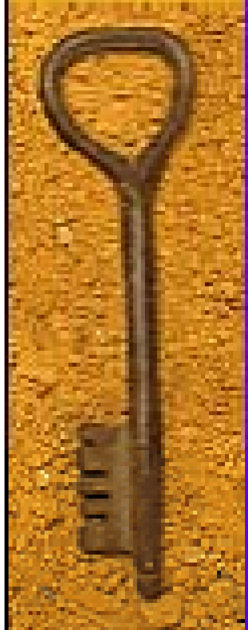
Horizontal orifice - circular or square opening in the flat surface of the bottom of the bin

Vertical orifice - opening in the side of the bin

Orifice characteristics affecting flow:

a) hydraulic diameter: 4A/P; A is the orifice area; P is the orifice perimeter

b) aspect ratio of rectangular orifice: ratio of the length of the longer sides to the length of shorter sides



c) **orifice size**: small orifice has a hydraulic diameter less than 15x the minor diameter of the particle; large orifice has a hydraulic diameter larger than 15x the minor diameter of the particle

$$Q = c_0 A D^n$$ ← mass flow regime

Q = volume flow rate, m^3/h

D = hydraulic radius of orifice, cm

A = area of orifice, cm^2

c_0 = constant, $m^3 cm^{-(n+2)} h^{-1}$

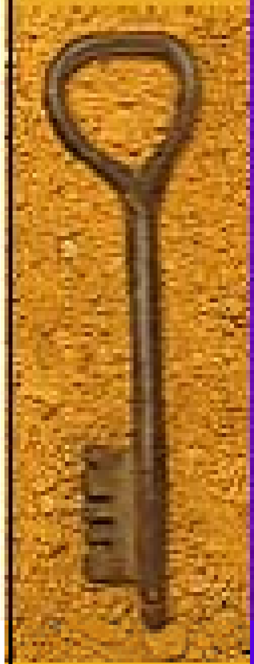
n = constant

Table 9.5. Values of Constants C and n for the equation predicting flow of grain through orifices.

Grain	Moisture Content (% w.b.)	Orifice Size (cm) Validated	Horizontal c	Horizontal n	Vertical c	Vertical n	Reference
Corn	12 to 15	13 to 25	0.0277	0.823	0.0155	0.791	Chang et al, 1984; 1990a
	20 to 22	13 to 25	0.0466	0.646	0.0185	0.702	Chang et al, 1984; 1990a
Wheat	13 to 15	10 to 25	0.0503	0.693	0.0380 [a]	0.542 [a]	Chang & Converse, 1988; Chang et al, 1990a
Sorghum	11 to 14	10 to 25	0.0922	0.461	0.0245 [b]	0.626 [b]	Chang & Converse, 1988; Chang et al, 1990a
	16 to 18	10 to 25	0.0784	0.532	-	-	Chang & Converse, 1988
Canola	6 to 12	7 to 20	0.055	0.7	-	-	Fast and Moysey, 1988
Flaxseed	4 to 13	7 to 20	0.0415	0.7	-	-	Fast and Moysey, 1988
Black Eyed Peas		4 to 8	0.0148	1.0	-	-	Gregory & Fedler, 1987
Soybeans	12	10 to 30	-	-	0.0182	0.730	Chang et al., 1990a

[a] Valid for 10 to 15% m.c. for vertical orifices.

[b] Valid for 12 to 18% m.c. for vertical orifices.



Example: Estimate the lateral and vertical forces at the bottom of the wall of a bin 9.1 m in diameter filled to a depth of 30 m with wheat at 11% m.c. The bin is made of concrete (wood float finish).

Example: Estimate the mass flow rate of wheat for a) horizontal and b) vertical orifice with a diameter of 20 cm.