Statistical distribution of physical attributes

- ✓Dimensions, weights, volumes are random quantities which follow a statistical distribution

- ✓Population: mean μ

  standard deviation σ

- ✓Sample: mean $\bar{x}$

  standard deviation s

- ✓Coefficient of variation $CV = \frac{s}{\bar{x}} 100$

Statistical Distribution of Physical Attributes

Physical attributes are random variables

f(x) – probability density function which describes the distribution of attributes

for random variable X to fall within a certain closed interval (a,b):

$$P\{a \le X \le b\} = \int_a^b f(x)dx \quad \longleftarrow \quad 1$$

for random variable X to be less than "a":

$$P\{X \le a\} = \int_{-\infty}^a f(x)dx \quad \longleftarrow \quad 2$$

Normal distribution – frequently-used probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2} \right) \qquad -\infty < x < \infty \quad \longleftarrow \quad 3$$

μ = mean value of random variable

σ = its standard deviation

x is normally distributed with mean μ and standard deviation σ

The random variable Z is normally distributed with mean 0 and standard deviation 1:

$$Z = \frac{X - \mu}{\sigma}$$

For variables of mean other than 0 and standard deviation other than 1, the probability that X lies within the range (a,b) is determined by calculating:

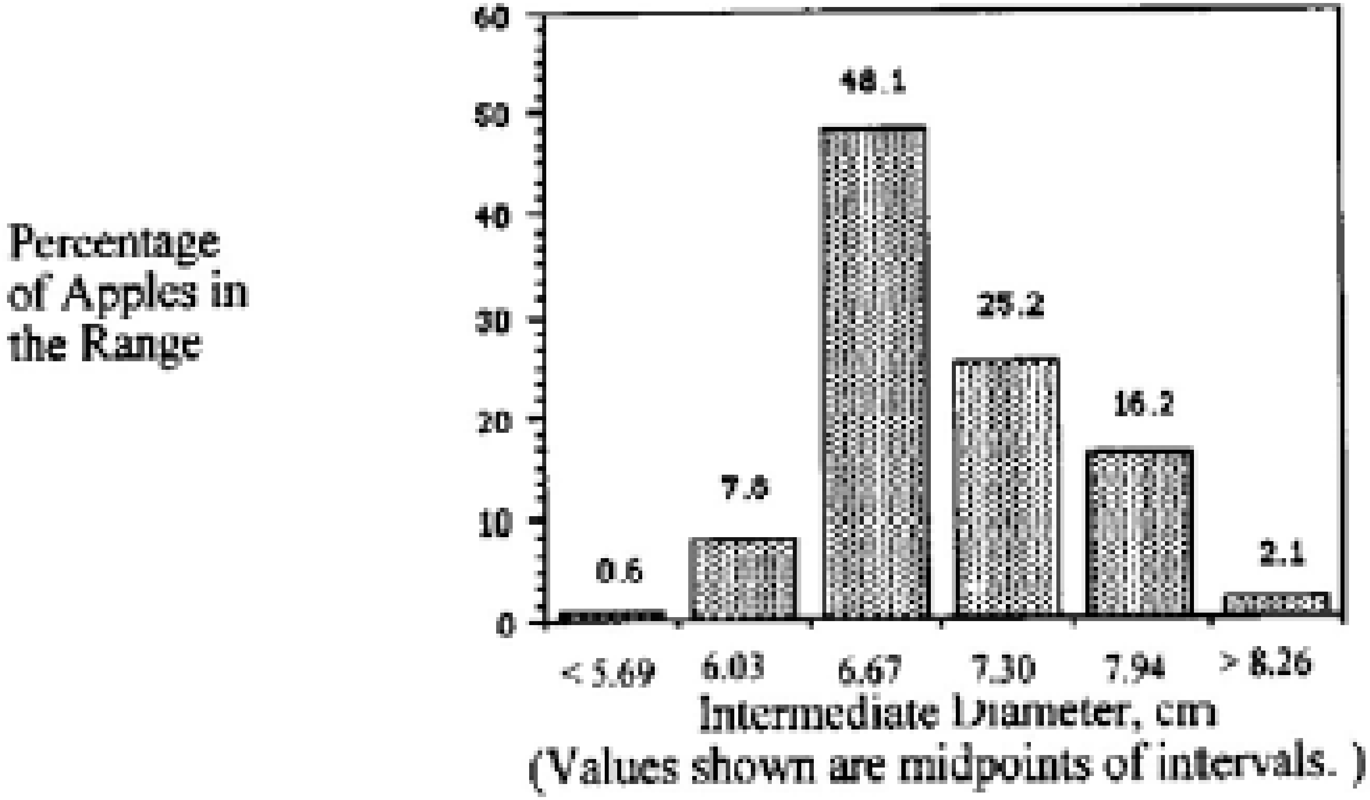
$$Z_1 = \frac{a - \mu}{\sigma}$$

and

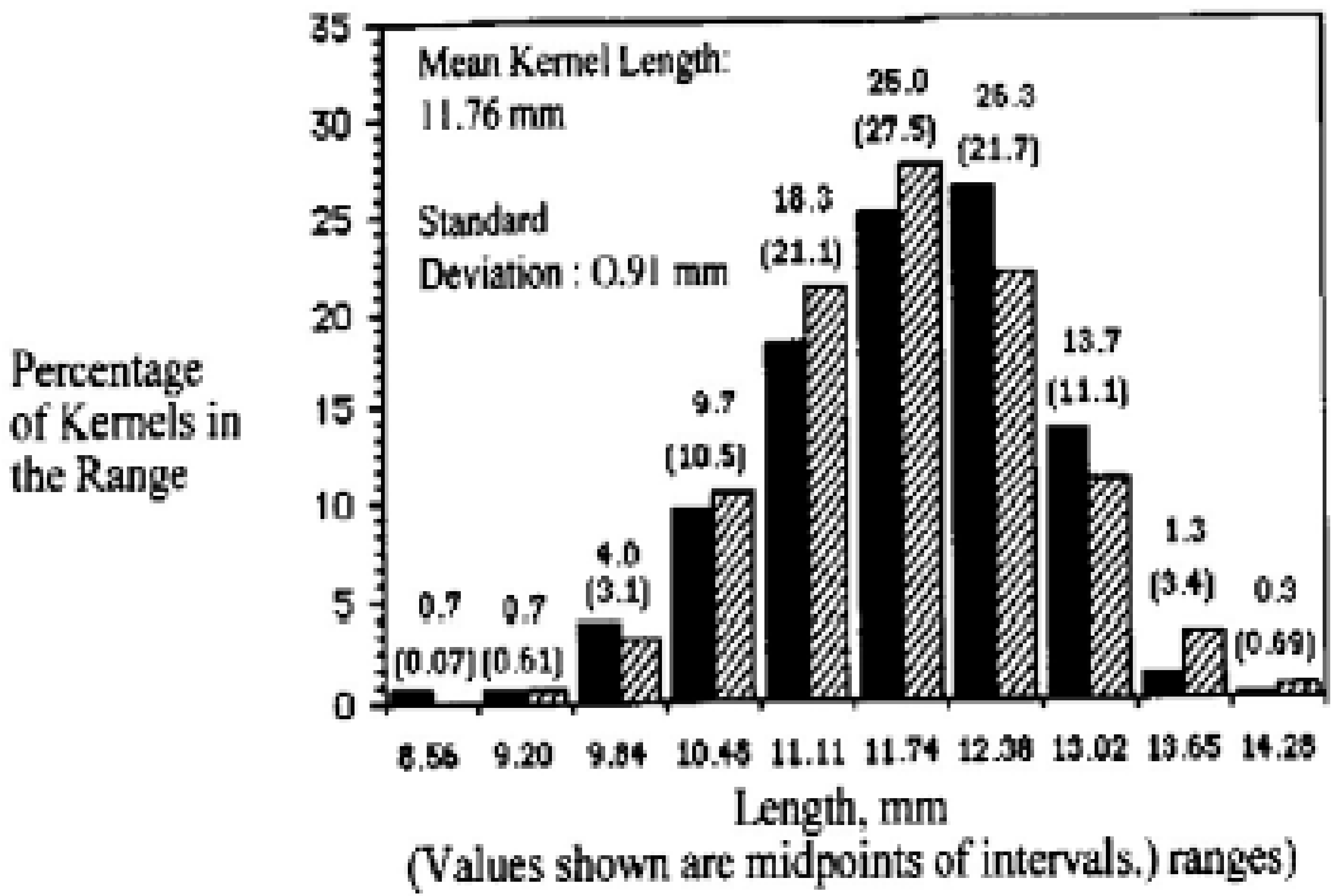
$$Z_2 = \frac{b - \mu}{\sigma}$$

Example 2.7. Determine the percentages of flat kernels (from the middle of the ears) of corn hybrid B73xM017 with length falling in the range of 10.16 to 10.79 mm. Assume kernel lengths follow a distribution similar to Fig. 2.3.

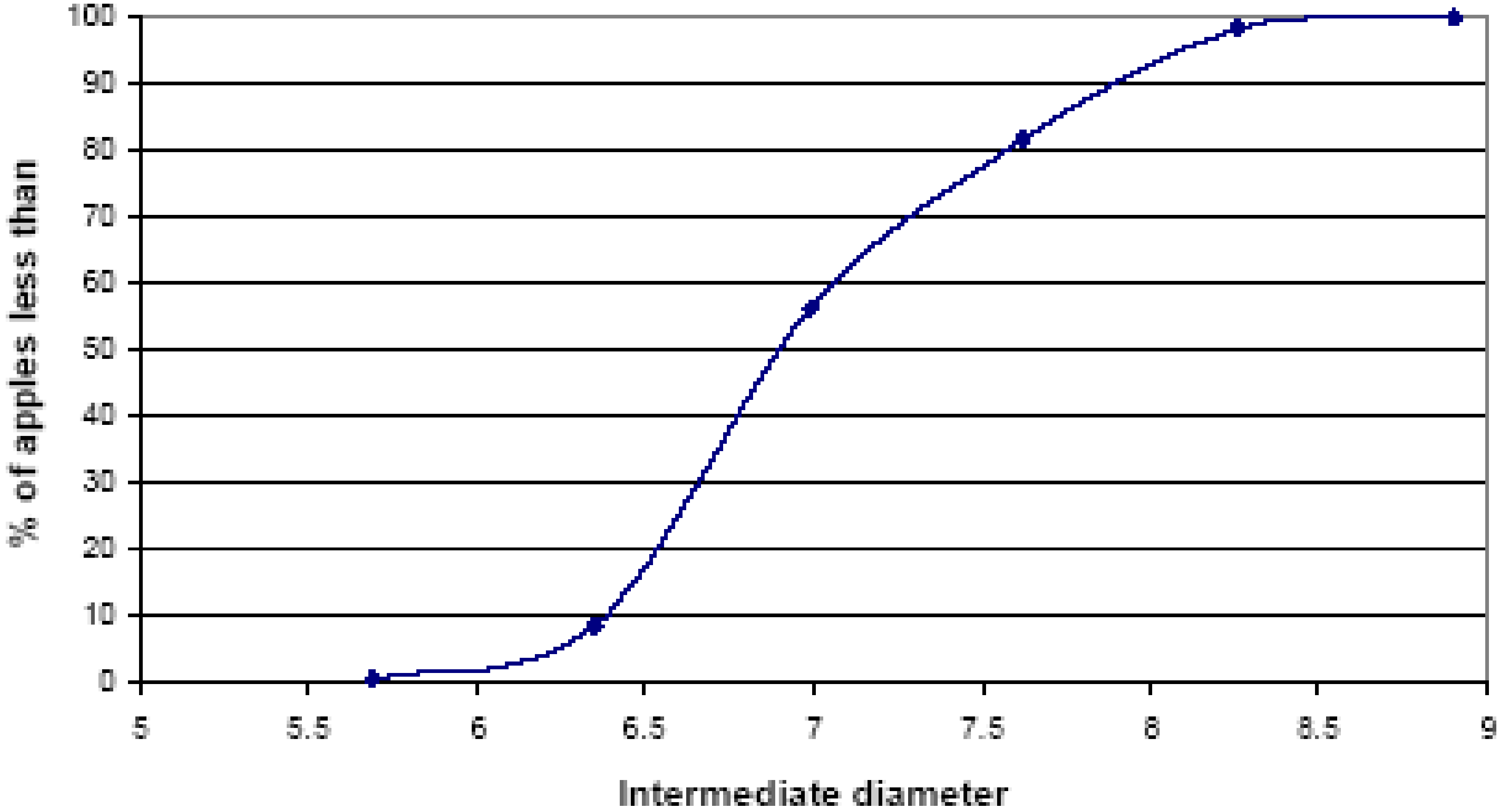
Below what length do 95% of the kernels fall?



If the attribute does not follow normal distribution, the numbers falling in a given range can still be estimated



Percentage of Kernels in the Range
Mean Kernel Length: 11.76 mm
Standard Deviation : 0.91 mm
0.7 (0.07)
0.7 (0.61)
4.0 (3.1)
9.7 (10.5)
18.3 (21.1)
25.0 (27.5)
26.3 (21.7)
13.7 (11.1)
1.3 (3.4)
0.3 (0.69)
0 5 10 15 20 25 30 35
8.56 9.20 9.84 10.48 11.11 11.74 12.38 13.02 13.65 14.28
Length, mm
(Values shown are midpoints of intervals.) ranges)



100
90
80
70
60
50
40
30
20
10
0
% of apples less than
5
5.5
6
6.5
7
7.5
8
8.5
9
Intermediate diameter

Particle Size distribution

For ground materials:

-increase mass transfer during extraction

-increase heat transfer

-increase digestibility

Probability density function for ground and spray-dried materials (log-normal distribution):

$$f(x) = \frac{1}{\ln\sigma(\sqrt{2\pi})x}\exp\left[-\left(\frac{\ln x - \ln\bar{x}}{(\ln\sigma)\sqrt{2}}\right)^2\right], x > 0$$

$\bar{x}$ = geometric mean of x

$$\bar{x} = \ln^{-1}\left(\frac{\sum_{i=1}^{m} N_i \ln(x_i)}{\sum_{i=1}^{m} N_i} \right)$$

s = geometric standard deviation

$$\sigma = \ln^{-1}\left(\frac{\sum_{i=1}^{m} N_i (\ln x_i - \ln \bar{x})^2}{\sum_{i=1}^{m} N_i} \right)^{\frac{1}{2}}$$

N_i = number of x's with value of x_i

m = the number of x_i's in the distribution

N_i = number of x's with value of x_i

m = the number of x_i's in the distribution

Weibull distribution - used in engineering practice to describe fatigue and failure of components

$$f(x) = \left(\frac{n}{x}\right)\left(\frac{x - x_0}{\bar{x}}\right)\exp\left[-\left(\frac{x - x_0}{\bar{x}}\right)^n\right], x \geq 0$$

f(x) = 0 when x < 0

$\bar{x}$ = a characteristic value of x (for particle size distribution it is a specific characteristic particle size) n = a constant characterizing the uniformity of x

x_0 = min. value of x or min. particle size

x = particle size

Particle weight is used in analysis and the cumulative weight of particles falling beneath a given size range x is:

$$W(x) = 1 - \exp\left[-\left(\frac{x - x_0}{\bar{x}} \right)^n \right]$$

for agricultural materials:

n = 0.5 - 1.3

$\bar{x}$ can be chosen so that 36.8% of the particles will pass through a sieve with diameter opening. (it is equivalent of e^{-1})

Techniques of particle size analysis

- Microscopic analysis
- Electronic resistivity (Coulter counter)
- Air elutriation
- Centrifuging
- Sedimentation
- Sieving - most common technique

  U.S. Std. Series or Tyler Series Sieves

  3 1/2 to #400

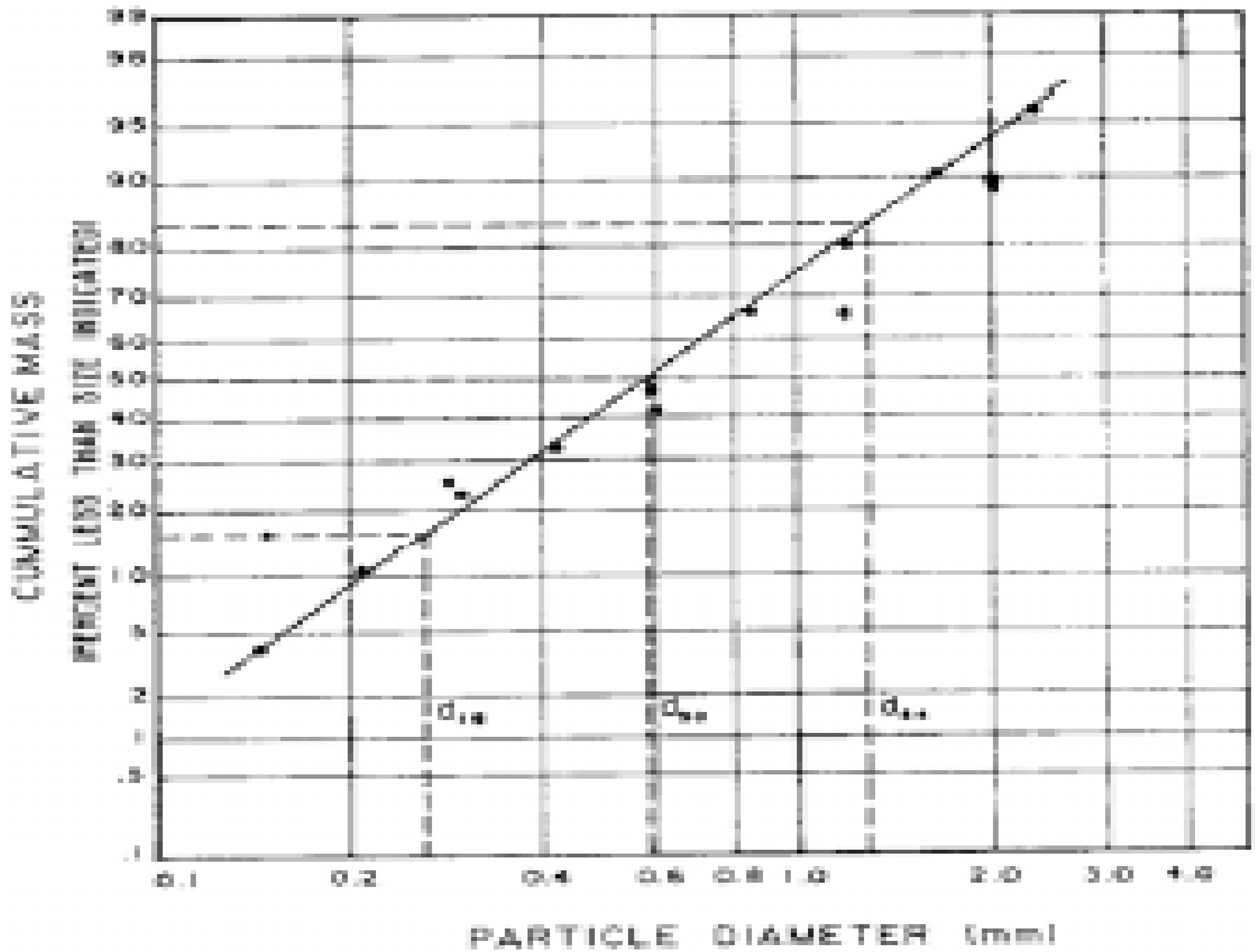


FIG. 2—CUMULATIVE PARTICLE SIZE DISTRIBUTION, BY MASS, FOR A GROUND CORN SAMPLE

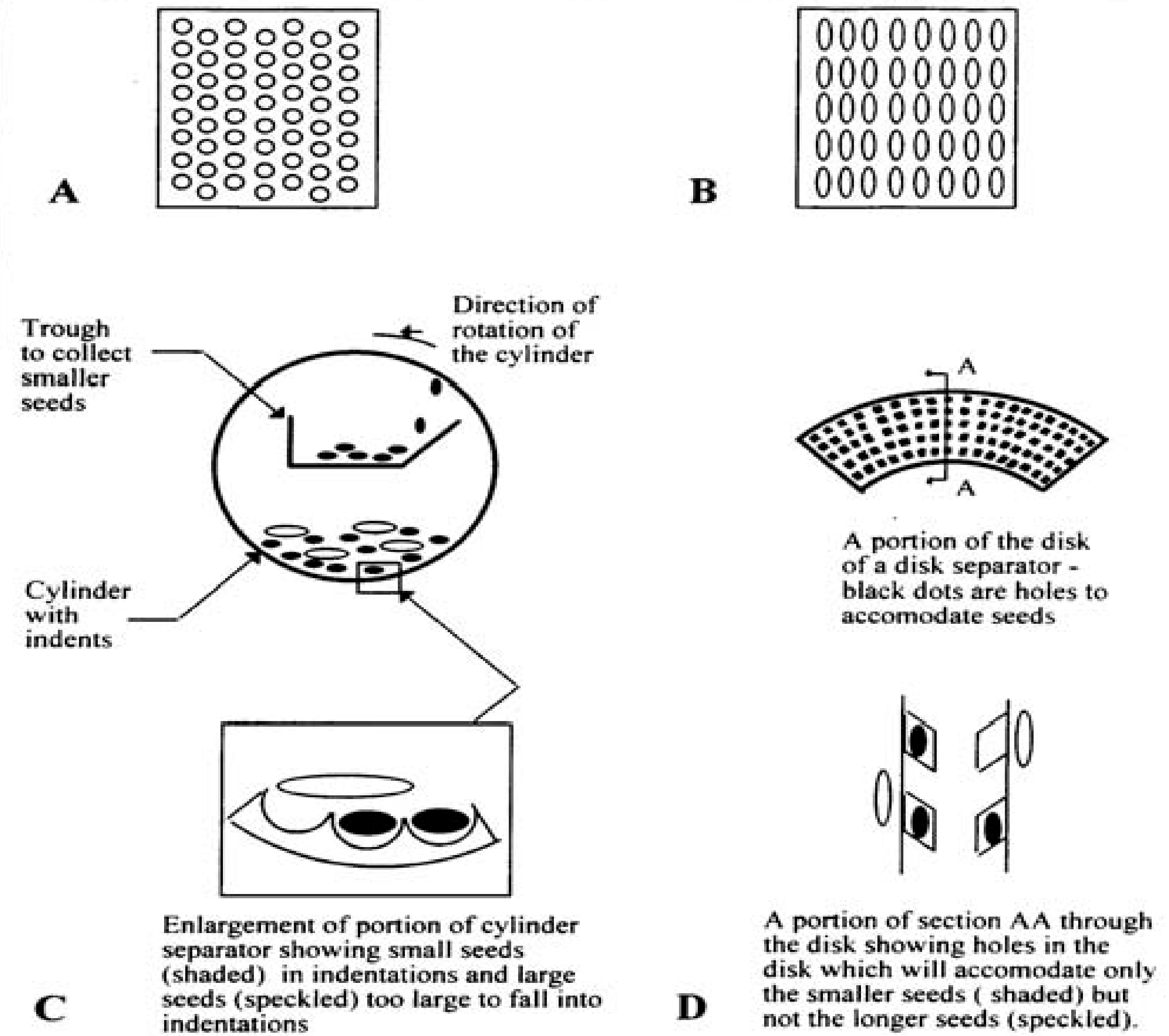
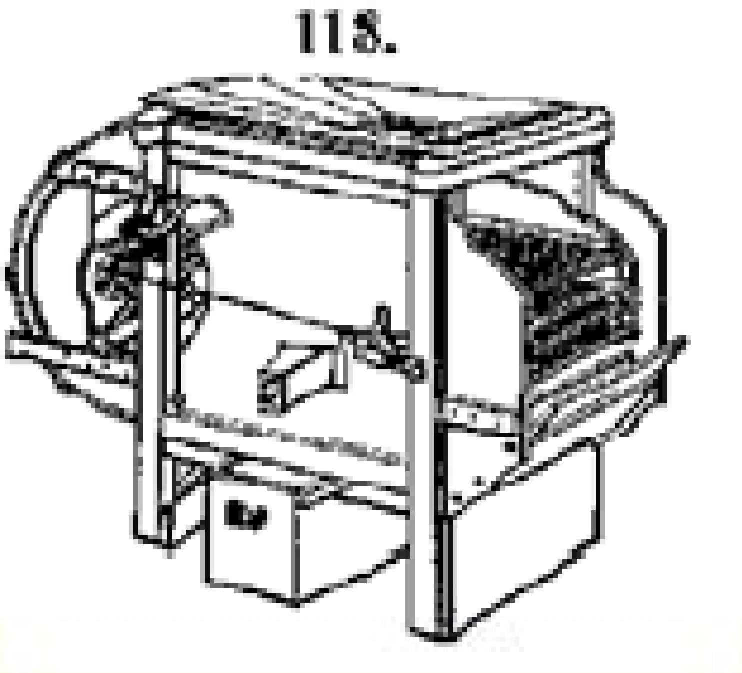


Figure 2.7. Devices used to separate seeds by their diameters: **A.** Round-hole sieve for intermediate diameter. **B.** Slotted sieve for minor diameter. **C.** Cylinder separator fo major diameter. **D.** Disc separator for major diameter.

Fanning Mill



Laboratory Indented Cylinder separator

Separation by shape

Spiral Separator

Separation by density

Lab. Gravity Table

Gravity Table

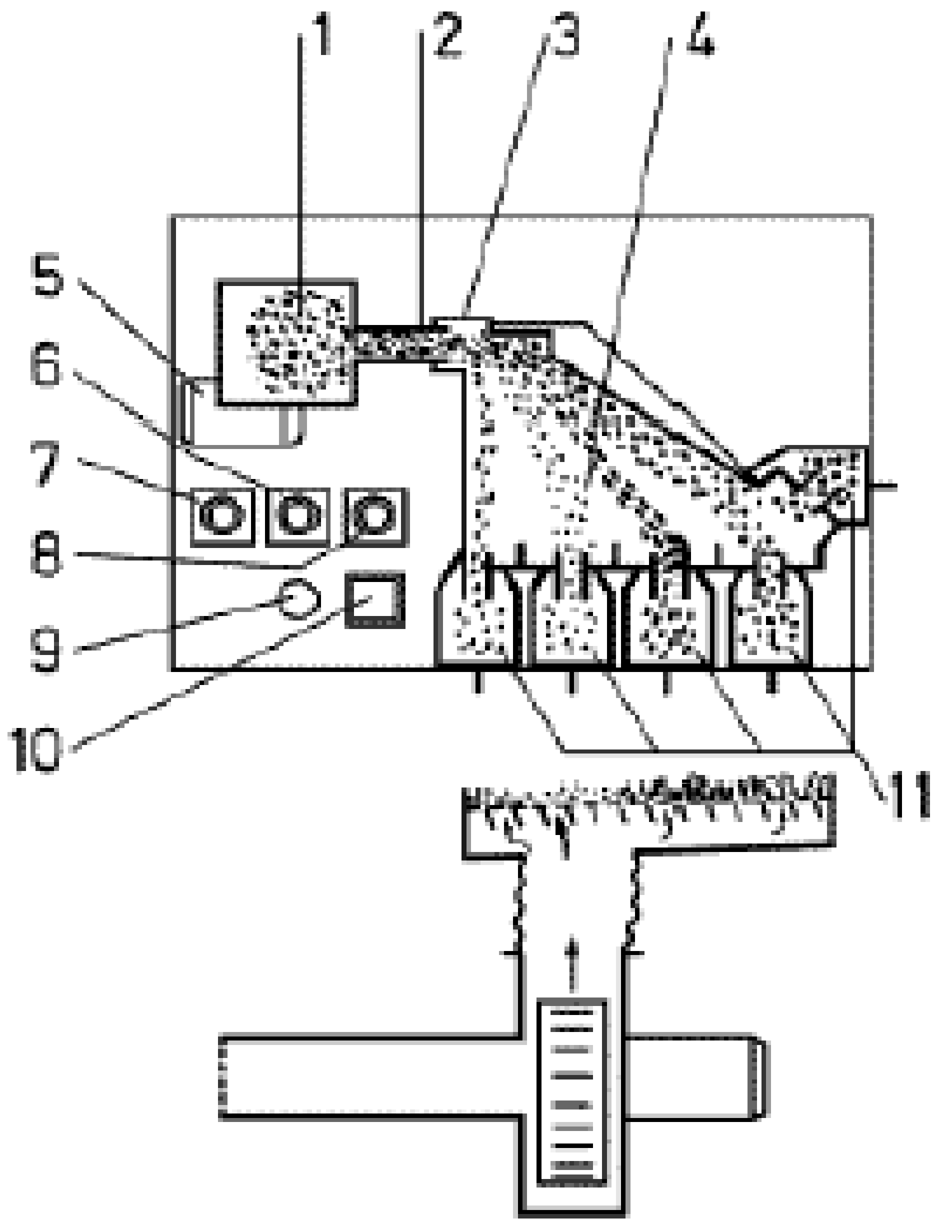
hopper (1)

vibrating chute (2),

inlet of the table (3)

table (4)

collecting drawers (11)



Aspirator

raw material (1)

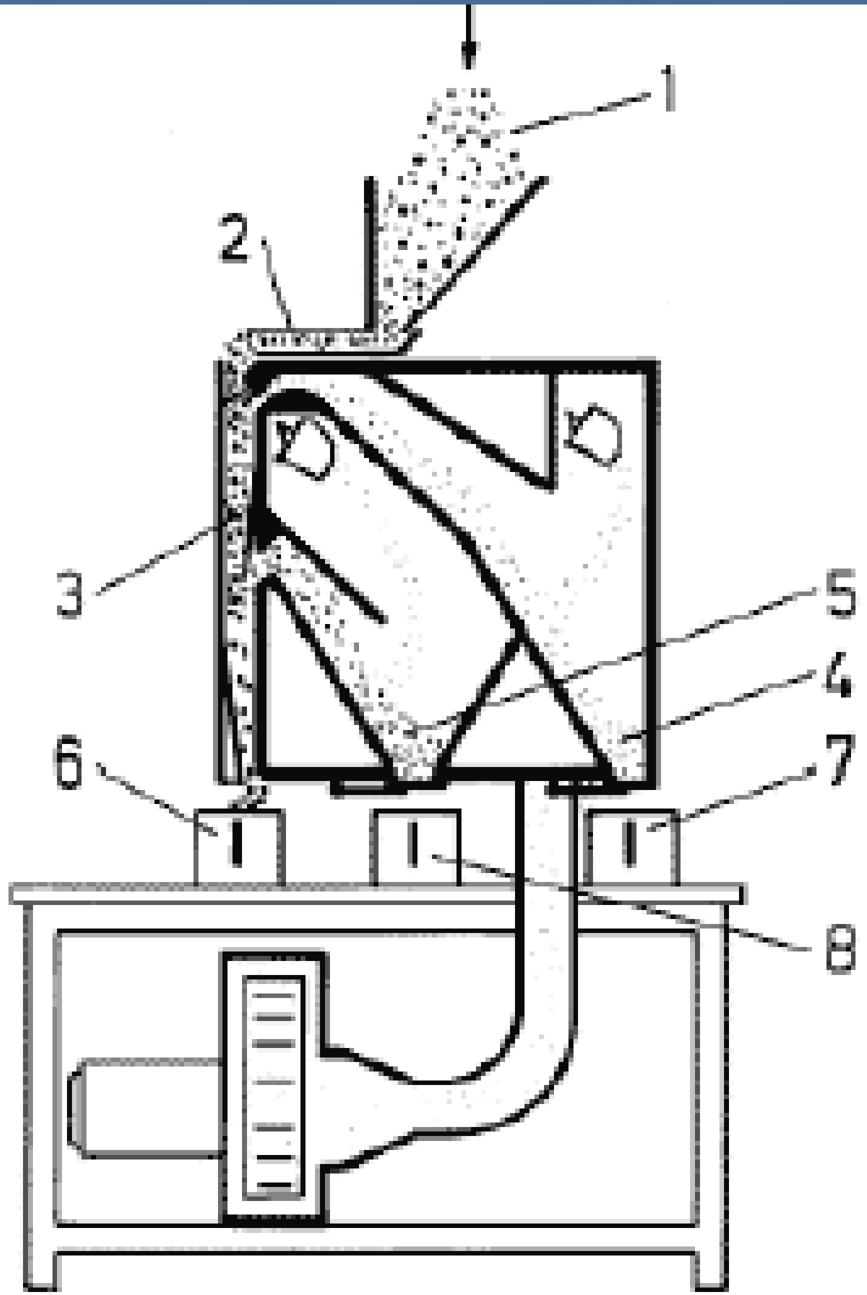
vibration feeder (2)

air separating chamber (3)

aspiration chamber 2 (4), whereas

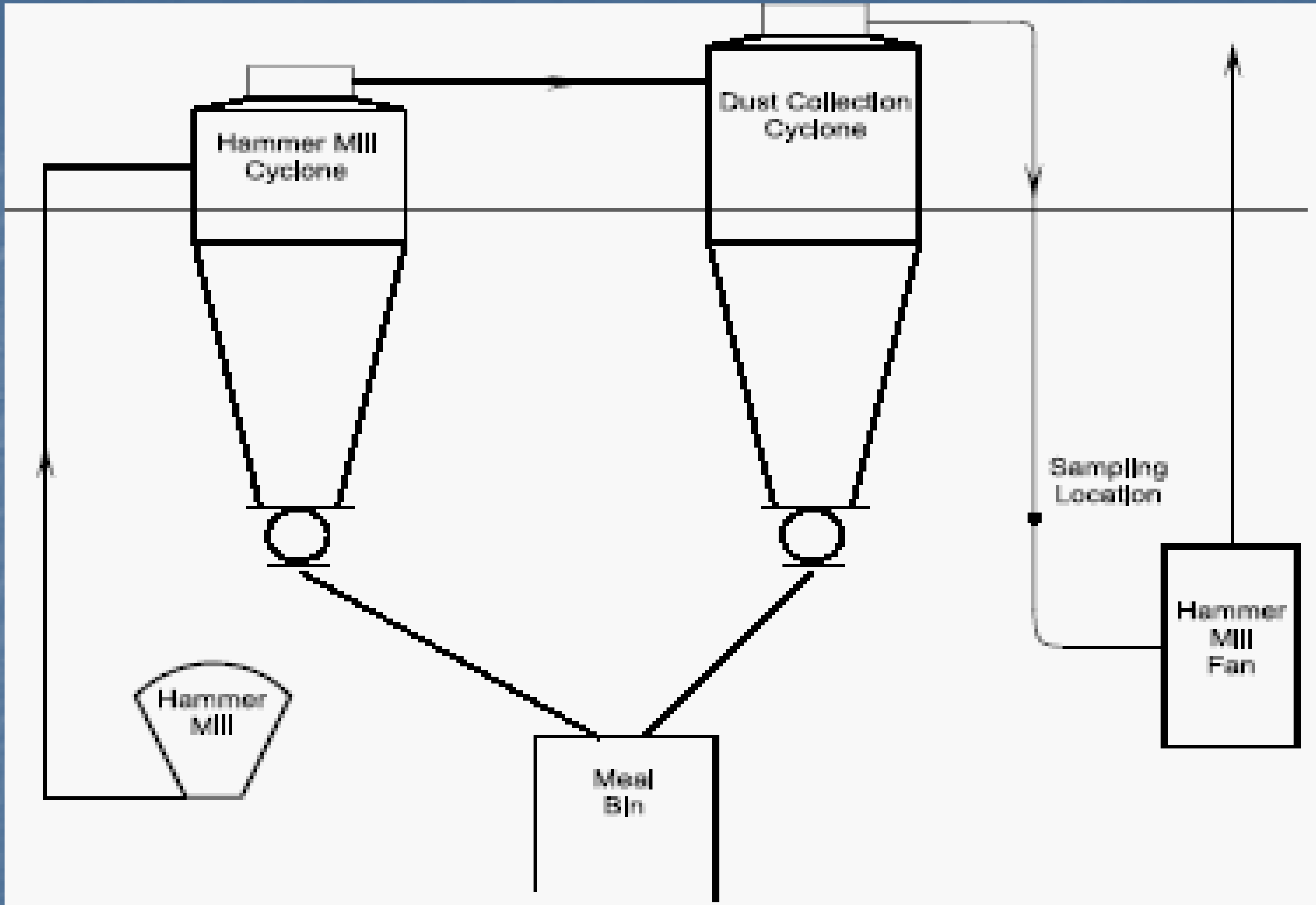
aspiration chamber 2 (5).

collecting box (6, 7, 8).



Manual cream separator

Dust collection cyclone installed in series with the hammer mill cyclone



Dust collection cyclone